Military Quartz Resonators

"AT" TYPE, 0.8-20 MC

CR-18/U and CR-19/U

Issued August 1961

PREPARED FOR
DEPARTMENT OF THE ARMY
UNITED STATES OF AMERICA

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400 South Jefferson Street, Chicago 7, Illinois

UNION THERMOELECTRIC DIVISION
COMPTOMETER CORPORATION
Niles, Illinois

THE DESIGN OF FUNDAMENTAL MODE THICKNESS-SHEAR QUARTZ RESONATORS

(MILITARY TYPES; CR-18/U and CR-19/U)

Written By

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Prepared For

THE DEPARTMENT OF THE ARMY

By

UNION THERMOELECTRIC DIVISION
COMPTOMETER CORPORATION
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ABSTRACT

This report covers that phase of the subject contract concerned with the design of fundamental mode quartz resonators in the CR-18/U frequency range. The work was performed during the period from 25 December, 1955 to 28 February, 1960.

The basic problems associated with AT type resonator design are discussed, along with the relative advantages of various plate configurations. In this study, investigations were limited to circular plates and to spherical beveling or contouring, where necessary.

Three plate designs were chosen for extensive study: plane parallel, plano-convex, and bi-convex. The effects of the various plate parameters on the performance of resonators of these designs are discussed. Tables of numerical results obtained with the experimental units that were fabricated are also included.

Satisfactory results were obtained with plane-parallel plates from 7 to 20 mc., with plano-convex plates from 1.4 to 9 mc., and with bi-convex plates from 850 kc. to 4.0 mc. Specific design data is given for plano-convex plates from 1.4 to 10 mc., and for bi-convex plates from 800 kc. to 1.5 mc.

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I. INTRODUCTION

A. Objectives

The purpose of this study was to provide a body of design information for fundamental mode quartz resonators of the military types CR-18/U and CR-19/U. At the time the program was started, the design information available in the literature was limited to specific frequencies or to narrow portions of the 800 kilocycle to 20 megacycle range. Design data for that portion of the frequency range below 3 megacycles was considered especially inadequate. The goal of this study was to provide, if possible, reliable design data for resonators of any frequency in the CR-18/U range.

In order to be of practical value, it was considered essential that the resulting design data meet certain requirements. The most important, of course, is that the performance specifications be met, preferably with sufficient safety margin to allow for normal manufacturing variations without incurring excessive rejection rates. A second requirement is that the designs be specified in such a manner that the results could be duplicated by others. To accomplish this, it was considered highly desirable that resonator designs be specified in terms of the actual physical dimensions of the plate, rather than by a description of the fabrication process followed. As an example, the two most common methods of generating a convex contour on the surfaces of a quartz plate are spherical lapping and tumbling in a cylindrical drum. The lapping method will produce a spherical surface on the plate with the same radius of curvature as the lap, if proper precautions are taken. The drum, on the other hand, usually produces a surface that has a roughly elliptical cross section; the actual dimensions depending upon the dimensions of the drum, the size of abrasive used, the speed of rotation of the drum, the amount of time in the drum, etc. Aside from the relative merits of the two plate configurations, the spherical contour is much more easily specified and more readily reproduced. A third requirement is that the designs chosen be economical to produce insofar as possible. This involves employing plate shapes that can be produced on existing equipment in industry, or at least equipment that is not prohibitively expensive. Also, from the economy standpoint, designs should be avoided which require "tailoring" each individual resonator by cut-and-try methods.

B. Scope of Investigations

A major consideration, in initiating a study of this nature, is the choice of the avenues of investigation to be explored. The goal of obtaining design data complete enough to cover the entire CR-18/U frequency range, together with the limitations of time and funds, made it necessary to restrict the number of variables as much as possible. The following points were adopted as general guidelines for the project:

(1) The performance characteristics the resonators would be required to meet was that prescribed by MIL-C-3098B. Reliability was considered of greater importance than achieving the best possible performance.

(2) The number of resonators having a given set of dimensions would be limited to the quantity required to determine the performance characteristics of that particular design. While this approach would not make it possible to obtain meaningful yield figures, it would provide a much broader body of data, resulting from the greater number of different designs that could be tested.

(3) The study would be restricted to circular plates, if possible, because of their obvious advantages over rectangular plates, at least from a fabrication standpoint.

(4) Uncontoured plates would be employed in all cases where the performance specifications could be reliably met by this design.

(5) In those cases where plane-parallel plates proved unsatisfactory, spherical shaping would be employed, either in the form of an edge bevel or a fully contoured surface.

(6) No extensive investigation of various types of mounting supports would be made. If possible, the quartz plate would be securely bonded to the supports, in order to avoid the undesirable effects associated with free-floating mounts during shock and vibration tests.

(7) Electrodes would be deposited by evaporation only, using the metals commonly employed. No final frequency adjustment would be performed, since precise control of the resonant frequency is unnecessary for a study of this type.

II. PRINCIPLES OF OSCILLATOR PLATE DESIGN

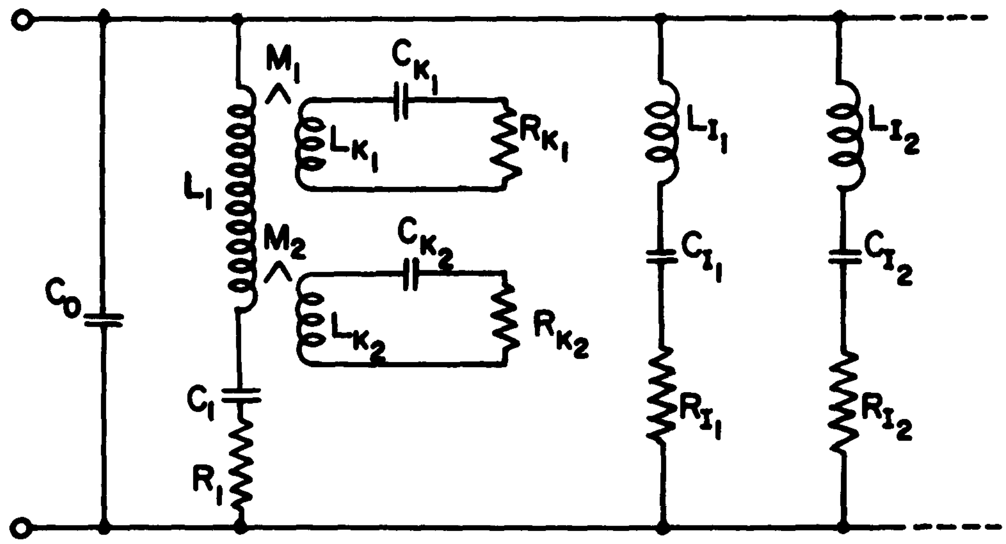
A. Influence of Unwanted Modes of Vibration

When an elastic body is in steady-state vibration, each particle of the body either remains motionless or moves periodically about its position of equilibrium. The amplitude of this motion will vary from point to point in the body producing a certain pattern of vibration. Generally speaking, each of the many possible vibration patterns is associated with a discrete resonant frequency and is referred to as a mode of vibration. With the exception of a few special types employing coupled modes of vibration, quartz resonators are intended to vibrate in a single mode. Unfortunately, this ideal situation is probably never quite achieved in practice. The influence of other modes of vibration on the performance of a quartz resonator can be seen by examining the simplified equivalent circuit of the resonator as shown in Figure 1. * C_o is the capacitance of the electrodes and the mounting supports; L_1, C_1, and R_1 are the equivalent motional parameters of the desired mode of vibration (the thickness-shear in the case of the AT type); L_K, C_K, and R_K are the parameters of mechanically coupled modes and L_I, C_I, and R_I are the parameters of the so-called inharmonic overtones.

* Parasitic Vibrations in Quartz Oscillator Plates, Project No. 90-408A, Report No. 11, Appendix IV, July 1, 1947, Armour Research Foundation.

FIGURE 1

EQUIVALENT CIRCUIT OF A QUARTZ RESONATOR WITH UNWANTED MODES OF VIBRATION



Fundamental Thickness-Shear Mode: L_1, C_1, R_1

Mechanically Coupled or Parasitic Modes: L_K, C_K, R_K

Inharmonic Thickness-Shear Overtones: L_I, C_I, R_I

Effective impedance added to fundamental mode by a parasitic:

$$Z' = \frac{\omega^2 M^2}{R_K + J\left(\omega L_K - \frac{1}{\omega C_K}\right)}$$

The equivalent resistance at coincidence of fundamental and parasitic resonant frequencies:

$$R_s = R_1 + \frac{\omega^2 M^2}{R_K}$$

The mechanically coupled or parasitic modes are of greater importance in the design of resonators for oscillator use, while the inharmonic overtones are of primary concern in resonators for filter use.

Mechanically Coupled or Parasitic Modes

Considering the main mode as the primary and a parasitic mode as the secondary of a coupled circuit, the effect of the coupled mode is equivalent to adding an impedance in series with the primary having the value given by Z' in Figure 1. When the resonant frequency of the secondary is identical to that of the primary, the effect of the parasitic mode on the main mode is purely resistive. The value of the motional resistance of the resonator is then given by the expression at the bottom of Figure 1.

From the equivalent circuit, we see that the effects of the coupled modes on the main mode can be reduced by any of the following techniques:

(1) Change the resonant frequency of the coupled mode so that it does not approach that of the main mode.

(2) Reduce the equivalent mutual inductance, M, between the main mode and the coupled mode.

(3) Increase the equivalent resistance, R_K, of the coupled mode.

All three of these techniques have been employed with some success, but of the three, only (2) has been of significant practical value.

There are a number of different types of mechanically coupled modes which can cause difficulty in AT resonators, among these are the flexure, contour shear, and contour extensional modes.

The relation of these modes to the thickness shear mode for uncontoured plates of small diameter to thickness ratio is shown in Figure 33, Appendix A. In addition, many modes have been observed which have not been identified. One of these is represented by Figure 34.

Inharmonic Overtones

Unlike the mechanically coupled modes, the inharmonic overtones car be excited independently of the main mode. These are thickness-shear modes similar to the main mode but with more complex vibration patterns. Fortunately these modes usually have resonant frequencies that are significantly higher than the main mode and hence cannot be excited simultaneously with it. The principal reason they are of concern in the design of resonators for oscillator use is that, under certain conditions, an inharmonic mode may have a lower impedance than the main mode, making it possible for the oscillator to shift frequency to that of the inharmonic overtone. This difficulty can be avoided by proper design of the resonator, either by increasing the impedance of the inharmonic mode or by increasing the frequency difference between the main mode and the inharmonics. However, these modes do present a major problem in the design of resonators for use in filters, since their impedance must be much higher than is required for satisfactory oscillator use.

B. Plate Dimensioning and Contouring

Uncontoured Plates

The most satisfactory design for an AT resonator from the standpoint of ease of fabrication, is the circular, plane-parallel plate. The resonant frequencies of the mechanically coupled modes of plates of this design are primarily determined by the diameter of the plate and to a lesser degree by its thickness. Since the main mode frequency (thickness-shear) is only slightly affected by changes in plate diameter, an obvious approach to control of the mechanically coupled modes is to adjust the plate diameter so that there is no unwanted mode near the main mode frequency.

In practice this is not feasible unless the resonator is only required to operate over a very narrow temperature range. The mechanically coupled modes have relatively high frequency-temperature coefficients and consequently change resonant frequency much more than the thickness shear mode over a given temperature range. Therefore, to avoid interference, the frequency spectrum must be free of parasitic modes over a much wider frequency range than the excursion which the main mode will experience.

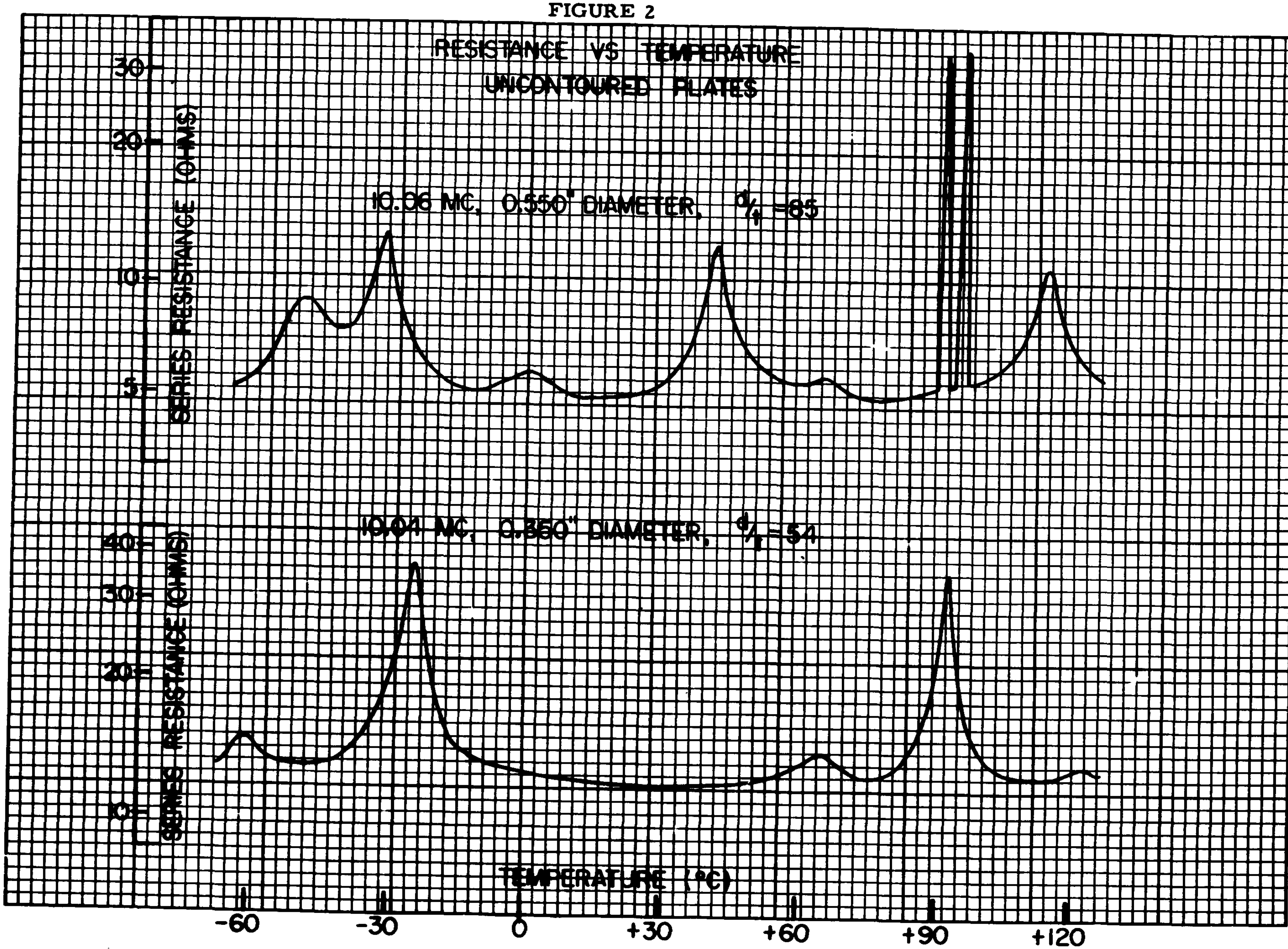
Plates having a small diameter to thickness ratio will have a more widely spaced parasitic mode spectrum than that of plates with a large ratio, as shown in Figure 2, but a small diameter to thickness ratio generally results in excessive damping of the main mode. Plane-parallel plates having large diameter to thickness ratios can be successfully used even though the parasitic mode spectrum is more densely populated, because the coupling to the main mode becomes weaker as higher overtones of the parasitic mode are encountered.

Beveled Plates

The damping of the main mode experienced by plates with small diameter to thickness ratios can be reduced by beveling the edge of the plate. This is usually done by shaping either one or both major surfaces of the plate on a concave spherical lap so that the thickness at the edge is less than in the center of the plate. The effect on the parasitic modes, in this case, is much more complicated than merely changing the plate diameter. Since the plate no longer has a uniform thickness, the boundary conditions for any mode that is strongly influenced by the lateral dimensions will be considerably altered. Instead of two parameters to consider (plate diameter and thickness) we now have four: plate diameter, diameter of the plateau (the flat portion remaining in the center of the plate), thickness at the center, and thickness at the edge (or radius of curvature of the beveled portion).

The effect on the parasitic mode influence that will result from beveling the plate will depend quantitatively on the interrelation of the four parameters mentioned above, but a few qualitative

FIGURE 2


RESISTANCE VS TEMPERATURE
UNCONTOURED PLATES
10.06 MC, 0.550" DIAMETER, d/t =85
10.04 MC, 0.350" DIAMETER, d/t =54
SERIES RESISTANCE (OHMS)
30
20
10
5
SERIES RESISTANCE (OHMS)
40
30
20
10
TEMPERATURE (°C)
-60
-30
0
+30
+60
+90
+120

statements can be made. If the diameter to thickness ratio is large (60 or greater), the influence of parasitic modes will not be appreciably reduced, even though their spacing is altered, unless a considerable portion of the surface is beveled. Fortunately, for plates of this d/t ratio the plane parallel design is usually satisfactory. Plates having intermediate values of diameter to thickness ratio (60 to 17) usually require some shaping of the major surfaces. Beveling can improve the performance of these plates if the proper combination of plateau diameter and bevel curvature are used, but determining these values by trial and error can be a very lengthy process. For plates having a diameter to thickness ratio of less than 17, it is generally necessary to bevel the major portion of the plate with a relatively high curvature. The effect of this is to make the curvature of the beveled portion the critical parameter and the diameter of the plateau of less importance. For practical purposes, the plate is then essentially identical to a fully contoured plate, both in regard to performance and fabrication problems.

Fully Contoured Plates

The influence of the parasitic modes on a thickness-shear resonator can be considerably altered by applying a spherical contour to either one or both major surfaces of the plate. There are actually three effects taking place simultaneously when a plate is contoured. Referring to the equivalent circuit of Figure 1, the three effects can be described as changing: (1) the resonant frequency of the parasitic modes, (2) the equivalent resistance of the parasitic modes, and (3) the effective coupling between the main mode and the parasitics. The last of these is probably of the greatest significance in oscillator plate design. For example, the extraordinary performance of plano-convex plates with the "optimum" curvature described in Section IV is apparently due to a minimizing of the coupling between the parasitic modes and the main modes.

The advantage of a fully contoured plate over a bevelled plate, from a fabrication standpoint, is the elimination of one of the physical parameters which is often critical, namely the plateau diameter.

Also, because of fabrication problems, the plano-convex design is to be preferred to the bi-convex, particularly in regard to maintaining the ZZ' orientation during the contouring operation.

Effects of Electrode Diameter

Since the lateral distribution of the electric field in the quartz plate will modify the thickness-shear strain distribution over the plate, we would expect the electrode size to have some influence on the coupling between the thickness-shear and parasitic modes. In this study no systematic investigation of this effect was made in connection with contoured plate design other than comparing the performance of a few different designs with various electrode sizes, but the indication is that the influence of electrode diameter on parasitic mode coupling is small compared to that of the plate dimensions.

C. Extrapolation of Design Parameters to a New Frequency

The ideal solution to the problem of quartz resonator design would be to have general design formulas for each type of plate. These formulas would provide all of the design parameters, i.e.: diameter, curvature, electrode size, orientation angle, etc., as a function of frequency. Thus it would be a simple matter to determine suitable parameters for any frequency in question. Such formulas have, in fact, been empirically derived for certain of the design parameters such as orientation angle, electrode thickness, etc. However, general formulas giving diameter and curvature for contoured plates are not so easily obtained. The reason for this is that the parasitic modes are so numerous and of such variety that complete systematic analysis is exceedingly difficult. Although we cannot predict the effect of each plate dimension on the relationship of each of the unwanted modes to the main mode, we can describe how the plate dimensions may be simultaneously changed in such a manner that these relationships will be maintained.

The Principle of Similarity

When a homogeneous, isotropic body vibrates in simple harmonic motion by virtue of its elasticity, the parameters that determine its resonant frequencies are the shape and size of the body, the elastic constants of the material, and the density of the material. Another body of the same material, having the same shape but of different size, will vibrate in the same modes of vibration, but the resonant frequencies of the modes will differ from those of the other body by the inverse ratio of their sizes.* When these conditions are met, two bodies are said to be geometrically similar.

For example, if we make two flat, circular plates of the same material, such that the diameter and the thickness of one are twice as large as the corresponding dimensions of the other, the frequency of each mode of vibration of the smaller plate will be twice the frequency of the corresponding mode in the larger plate. The truth of this statement is obvious when applied to a mode of vibration whose frequency is essentially controlled by a single dimension, such as the thickness modes in a plate of large diameter-to-thickness ratio. Its applicability is not restricted to these simple cases, however.

As an illustration, the equation for flexural vibration of a bar of length l and thickness e is**

$$f = \frac{e}{l^2} \frac{m^2}{4\pi\sqrt{3}} \sqrt{\frac{Y}{\rho}}$$

* Rayleigh, The Theory of Sound, 2nd Edition, Vol.2, p. 429, Dover, 1945.

** W.G. Cady, Piezoelectricity, p. 112, McGraw-Hill, 1946 where m is a coefficient depending on the order of the mode, Y is Young's modulus, and ρ is the density of the material. Since e and l are the only terms of the equation which are affected when we change the dimensions of the bar by some common factor, the frequency will be changed by the reciprocal of that factor, as required by the principle of similarity.

To apply the principle of similarity to quartz resonators, it is necessary to stipulate that the two resonators are not only geometrically similar but also have the same orientation with respect to the crystallographic axes. Otherwise, the elastic constants and the coupling coefficients between various modes of vibration would not be equal for the two plates.

Essentially, what the Principle of Similarity tells us is how to make resonators at different frequencies that will have their unwanted modes of vibration in the same relative relationships to their main modes. If we can then find a resonator design that is relatively free of unwanted modes near its main mode, we can make similar resonators at other frequencies that are equally free of unwanted modes. For example, if we have found a good plano-convex design for some frequency f_o, the dimensions for a similar plate at a new frequency f_1 will be:

$$\text{new thickness} = \frac{f_o}{f_1} \times \text{original thickness}$$

$$\text{new diameter} = \frac{f_o}{f_1} \times \text{original diameter}$$

$$\text{new radius of curvature} = \frac{f_o}{f_1} \times \text{original radius}$$

or when using diopters

$$\text{new diopter number} = \frac{f_1}{f_o} \times \text{original diopter number}$$

$$\text{new electrode dia.} = \frac{f_o}{f_1} \times \text{original electrode diameter.}$$

For economy of fabrication it is desirable to limit the number of different plate diameters, electrode diameters, and spherical laps used wherever possible. Strictly speaking, to maintain the same frequency relationship of the parasitic modes to the main mode, we must change all of the dimensions of the resonator when we change the main mode frequency. However, in certain cases, the parasitic modes that are of primary concern are affected much less by variations of the plate diameter, for instance, than by variations of the curvature of the contoured surface. When this is true, we can get essentially the same results by changing only the "critical" parameters and maintaining the same plate diameter, provided our frequency change is not too great. Unfortunately, as the diameter to thickness ratio becomes smaller all of the dimensions of the plate tend to become more critical, and we have less freedom in the choice of plate dimensions.

III. FABRICATION AND MEASUREMENT TECHNIQUES

The fabrication processes employed to produce the resonators used in this study were, in most respects, similar to those employed by the industry in general. Only those aspects having special significance to this project will be discussed here. More detailed information on fabrication and measurement techniques may be obtained from previous reports on the contract.

Final Lapping

The effects of various degrees of surface finish on the performance of a quartz resonator were not investigated extensively in this study. For the most part, at frequencies of 10 megacycles and above, 5 micron aluminum oxide was used as the final lapping abrasive. Below 10 megacycles, 15 micron silicon carbide was generally used. A few tests were made with polished uncontoured and plano-convex plates. They did not show any measurable improvement resulting from the polish, insofar as motional resistance is concerned.

Rounding

The tolerance imposed on plate diameter throughout this study was $\pm$ .001 inch. The rounding operation was performed on a cylindrical grinder using a 320 grit, metal bonded diamond wheel.

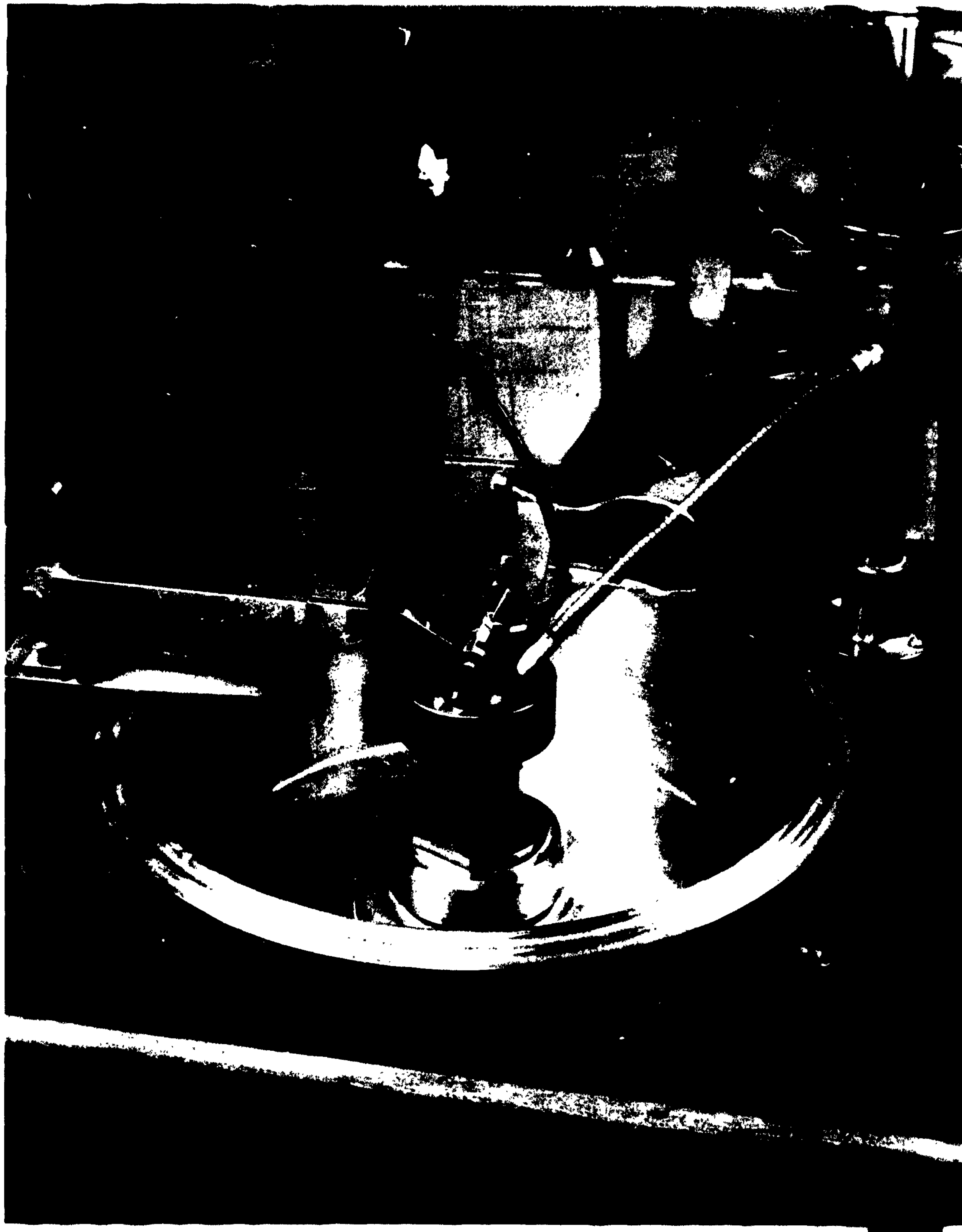
Spherical Contouring

Figure 3 is a photograph of an Elgin lap used to produce spherical surfaces on quartz plates. The arm attached to the left side of the bowl is a modification designed for use with laps of curvatures higher than 8 diopters. The oiler on the right side of the bowl holds the abrasive suspension which is fed through the flexible tube to the lap. The coaxial cable is connected to a radio receiver with an automatic shut-off device to stop the lapping operation when the desired resonant frequency is reached. The two metal buttons in the foreground show the two sides of the holders used to support the quartz plate during the contouring operation. The holders shown are used for plano-convex plates or for the first side of bi-convex plates. To contour the second side of a bi-convex plate, a similar holder with a concave recess of the same curvature is used.

For high curvature values, the lapping operation takes an excessive amount of time. The tool and cutter grinder shown in Figure 4 was adapted for use in rough contouring, as a means of reducing the lapping time. A vacuum chuck has been mounted in the work holder to hold the quartz plate. The motor attached to the work holder rotates the plate during grinding. A 400 grit, resinoid bonded diamond wheel is used. The metal shield covering the diamond wheel and the attached hoses provide for the coolant supply and return. An accurate spherical surface can be produced by a grinder of this type but the surface finish is not adequate without subsequent finishing on a spherical lap.

Electrode Deposition

The only method of electrode deposition employed in this study was vapor plating. The plating metal was evaporated at a pressure of 0.1 micron or less. Silver was used primarily, because of its ease in handling. The amount of metal applied was chosen to give a frequency change of approximately twice the minimum value shown in Figure 15. This amount was considered sufficient to allow for any variations in the plating operation which might result in an excessively thin film if the minimum value were used.



PIVOTING FIXTURE
ACCESSORY TO ELGIN LAP
CAE-1A

GORTON GRINDER WITH ACCESSORIES
CMG-1A

Mounting

Most of the quartz plates fabricated for this study were mounted in either of two types of mounting supports. These are the tab-clip and the RCA "rigid" types shown in Figure 5. At the top of the same figure, quartz plates of two different sizes (.375" and .550") are shown mounted in the tab-clip mount. The RCA mount was used for frequencies below 2.0 megacycles and the tab-clip for those above.

The small flat portion at the top edge of the larger plate in Figure 5 is used to determine mounting orientation. A segment of the plate from 3 to 5 thousandths of an inch deep is ground off, parallel to the mounting axis (usually Z'). On plates to be contoured, a similar segment is removed on the opposite side of the plate, to reduce any tendency for the plate to shift its orientation angle during contouring, as a result of being unequally supported.

The cement used to provide electrical and mechanical contact to the mounting supports is an epoxy known as Bondmaster M-640, impregnated with finely divided silver. The mixing and curing directions for this cement are as follows:

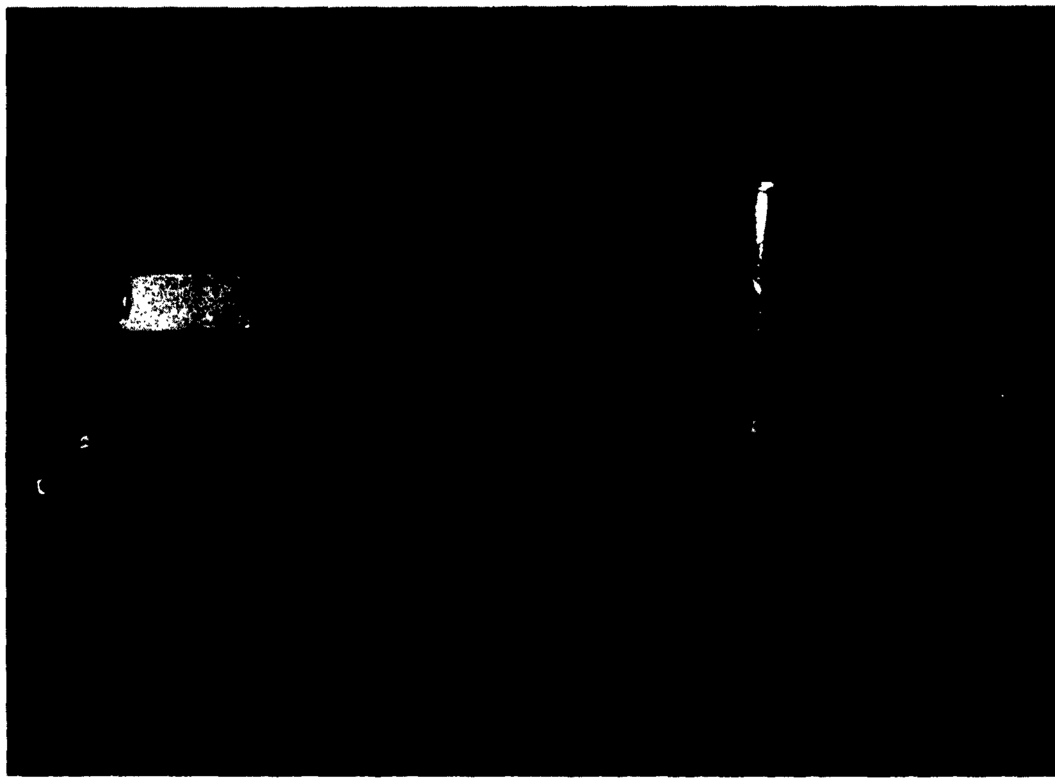
2.5 grams of Bondmaster M-640,	Rubber and Asbestos Corp. 225 Belleview Avenue Bloomfield, N. J.
7.5 grams of "Silflake #131,"	Handy and Harman, Bridgeport, Connecticut

Mix thoroughly in mortar with pestle for about five minutes.

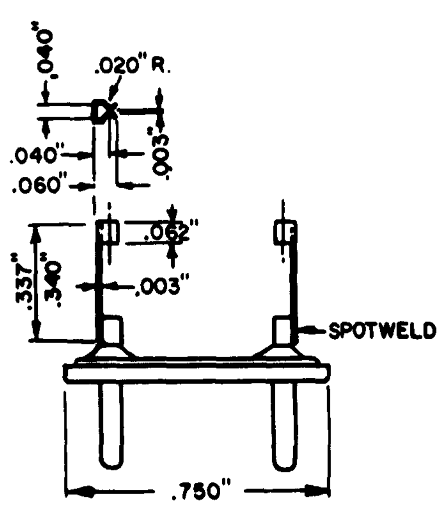
1.8 to 1.9 ml "Cellosolve Acetate" (ethylene gylcol monoethyl ether acetate), Carbide & Carbon Chemicals Co.
230 N. Michigan Avenue
Chicago, Illinois

Add in a mixing dish by means of a pipette. Stir until mixture reaches a creamy consistency, satin appearance. Do not use if it appears granular or sandy. Very small amounts of solvent may be added if necessary to achieve the proper consistency.

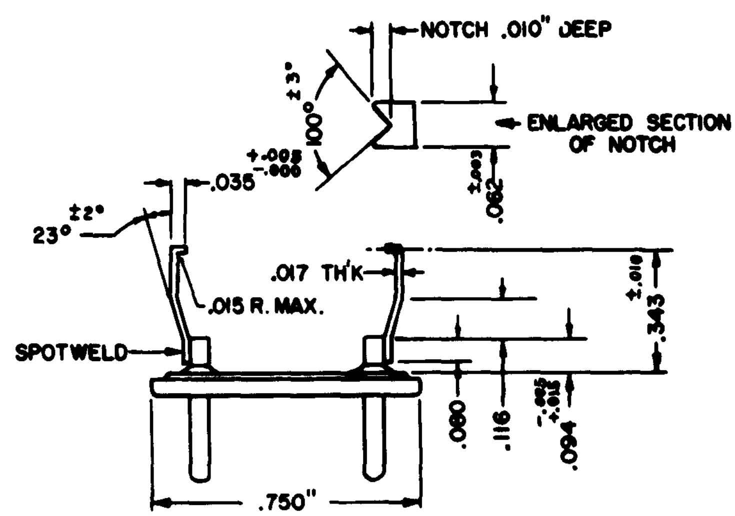
FIGURE 5



QUARTZ PLATES MOUNTED IN HC-6/U HOLDER



TAB CLIP MOUNT



R.C.A. "RIGID" OR "CLAW" TYPE MOUNT

Curing cycle of bond: 1-1/2 hours at 175°C.

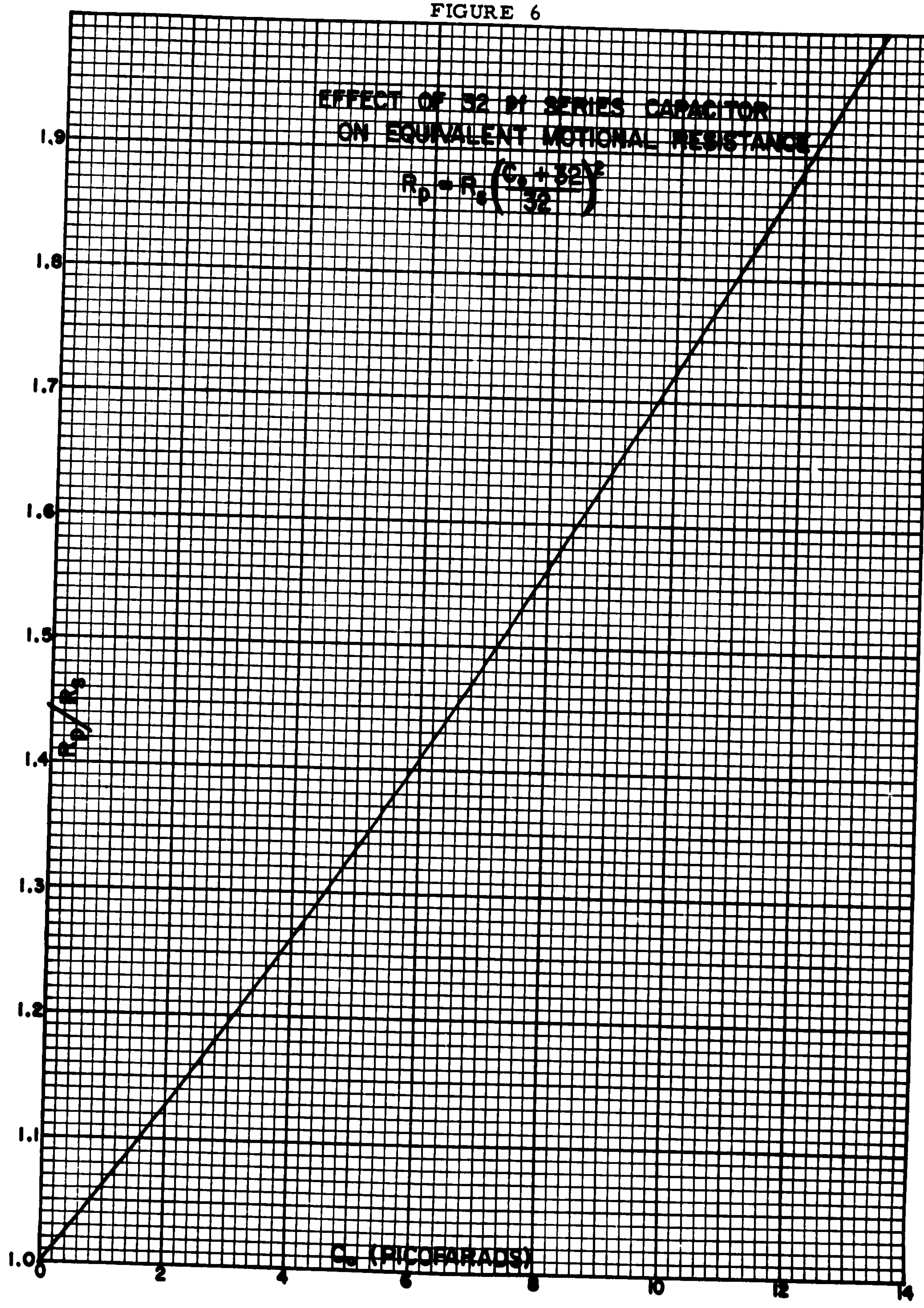
The mixture is kept refrigerated. More than 30 days refrigerated storage is not recommended.

Performance Measurements

The performance characteristics that were of greatest concern in this study were the equivalent resistance of the resonator and the variation of equivalent resistance and resonant frequency over the prescribed temperature range. The methods used to measure these characteristics were similar to those commonly employed in the industry with the exception that the "temperature run" was much slower than is customary. To achieve reasonable accuracy in the calibration of a dynamic temperature run (where the temperature is varying continuously) it is necessary to limit the rate of change of temperature. In this case the -55°C to +90°C range was covered in about 15 minutes, which is approximately 10°C per minute. A rate of change much higher than this is likely to cause errors in the temperature calibration and, in extreme cases, will create temperature gradients in the quartz plate itself, which may make the frequency and resistance measurements erroneous. For precise measurements of frequency versus temperature, a "stabilized" temperature run was performed. In this case, an "oven" was used which could be set and controlled at any temperature within the range. The frequency of the resonator was usually measured at 5°C intervals. After each change in oven temperature, sufficient time was allowed for the temperature of the resonator to stabilize.

During temperature tests, the resonators were operated at the drive level specified by MIL-C-3098B for that particular frequency. The holders of resonators under test were either unsealed or filled with nitrogen at a pressure of one atmosphere. Temperature runs were generally made with the resonator operating at series resonance rather than with a 32pf capacitor in series with the resonator. The conversion factor for the equivalent resistance with the series capacitor is given in Figure 6. A continuous recording of either equivalent resistance or frequency deviation, as a function of temperature, was made on an x-y graphic recorder. Two examples of actual temperature runs (reduced in scale) are shown in Figure 2.

FIGURE 6



EFFECT OF 32 pf SERIES CAPACITOR
ON EQUIVALENT MOTIONAL RESISTANCE
$R_p = R_s\left(\frac{C_o+32}{32}\right)^2$
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
R_p/R_s
0
2
4
6
8
10
12
14
C_o (PICOFARADS)

IV. EXPERIMENTAL RESULTS

A. Uncontoured Plates

Frequency-Temperature Characteristics

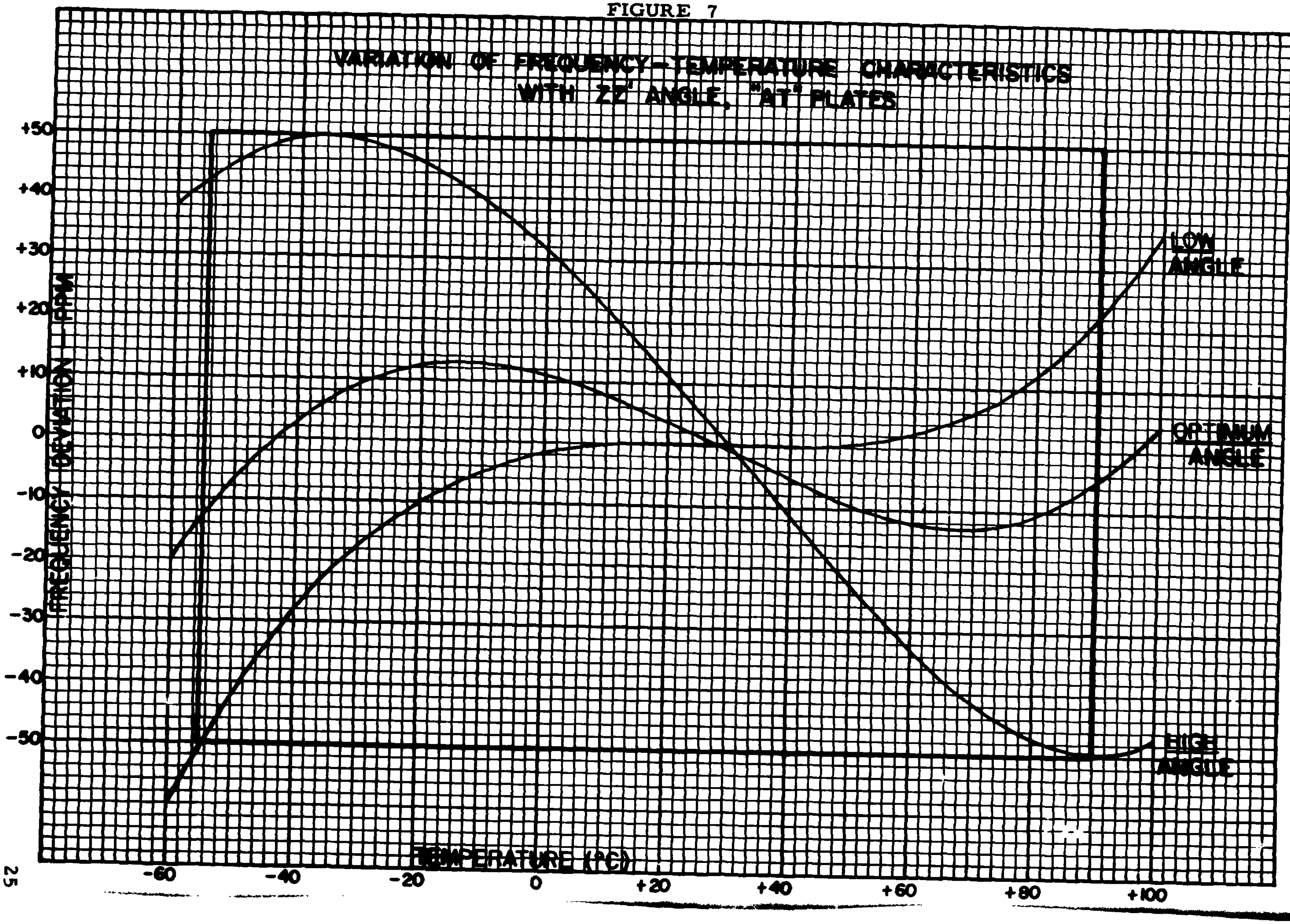
The maximum deviation of the resonant frequency from the nominal value allowed by MIL-C-3098B specifications is ±.005% or 50 parts per million. The frequency-temperature curves of Figure 7 were obtained with resonators having three different ZZ' orientation angles. The curve labeled optimum angle represents the minimum frequency deviation that can be obtained with an AT plate for the temperature range -55° to $+90^{\circ}$C. The total frequency excursion is about 26 parts per million or a deviation of ±13 parts per million from the nominal frequency. The low angle and high angle curves represent the extreme values of ZZ' angle which will meet the above specifications, assuming that the resonant frequency is adjusted to the nominal value at about 30°C. The angular difference between the "low" and "high" angles is about 15 minutes. The "optimum" angle is about 6 minutes above the "low" angle and about 9 minutes below the "high" angle.

The actual value of the optimum ZZ' angle will vary somewhat with the physical dimensions of the quartz plate and its electrodes, but the variation is much less for uncontoured plates than for contoured plates. The optimum angle for the uncontoured plates described in this report is about 35° 18'.

Motional Resistance

From the equivalent circuit of Figure 1, we see that there are two components of the series resistance of a resonator when a parasitic

FIGURE 7



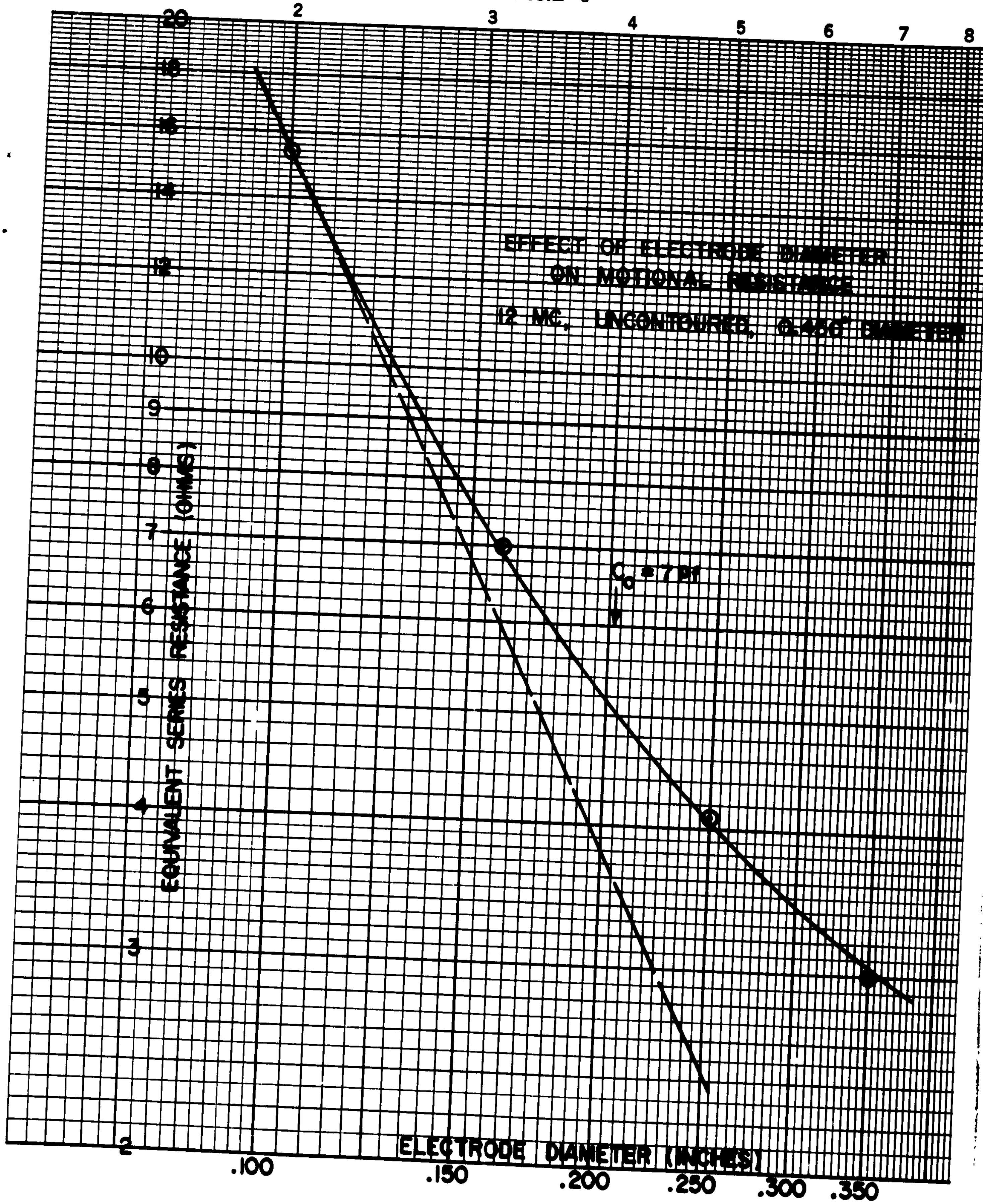
VARIATION OF FREQUENCY-TEMPERATURE CHARACTERISTICS
WITH ZZ' ANGLE, "AT" PLATES
LOW ANGLE
OPTIMUM ANGLE
HIGH ANGLE
FREQUENCY DEVIATION - PPM
TEMPERATURE (°C)
+50
+40
+30
+20
+10
0
-10
-20
-30
-40
-50
-60
-40
-20
0
+20
+40
+60
+80
+100

mode is being excited. These include what we might call the basic resistance of the main mode (R_1) and the effective resistance added by the parasitic mode $\left(\frac{\omega^2 M^2}{R_K}\right)$. We could further analyze the basic resistance (R_1) into various components such as metallic losses, atmospheric losses, internal friction, etc. However, these components will be essentially constant for a given resonator, at least over the temperature range in which we are interested. Unless the parasitic modes are so close together that the main mode is always under the influence of at least one of them, the resistance of the resonator will vary between the value R_1 and the sum of R_1 and the effective parasitic mode resistances as in Figure 2. We can assume, generally, that the minimum value of the equivalent resistance over the temperature range is equal to the "basic" resistance R_1.

The basic resistance of a resonator which employs an uncontoured plate depends upon many factors. Assuming that the parallelism and surface finish are adequate, the most significant variables are probably the plate diameter, the electrode diameter, and the mounting conditions. The effect of the electrode diameter, at least to a first order of approximation, is relatively simple. From an equivalent circuit standpoint, the motional resistance of a piezoelectric resonator is the ratio of the applied voltage to the piezoelectric current at resonance. Theoretically, if all other factors are held constant, the motional resistance should vary inversely as the square of the electrode diameter. Figure 8 is a graph of the average basic resistance of 4 groups of 12 MC, .450 inch, uncontoured units, each group having a different electrode diameter. For the resistance to be inversely proportional to the square of the electrode diameter, all of the points should lie on a straight line with a slope of minus 2, as shown by the broken line. The deviation from a straight line is largely due to the metallic resistance and the air loading resistance.

The effect of the plate diameter on the basic resistance of an uncontoured resonator is more difficult to predict than the effect of the electrode diameter. The principal factor here is the loss of vibration energy to the mounting supports. As the diameter to thickness ratio of the plate decreases, the amplitude of vibration at the edges of the plate becomes greater. Consequently, more energy is lost to the mounting supports and the motional resistance increases. To illustrate the effect of the plate diameter on the basic

FIGURE 8


EFFECT OF ELECTRODE DIAMETER
ON MOTIONAL RESISTANCE
12 MC, UNCONTOURED, 0.450" DIAMETER
EQUIVALENT SERIES RESISTANCE (OHMS)
ELECTRODE DIAMETER (INCHES)
.100
.150
.200
.250
.300
.350

motional resistance, in Figure 9 the lowest resistance values that were obtained with plates of various diameters and frequencies are plotted as a function of diameter to thickness ratio. All of the units of a given frequency have the same electrode diameter and for each frequency the electrode diameter is such that the shunt capacitance is approximately 7 picofarads.

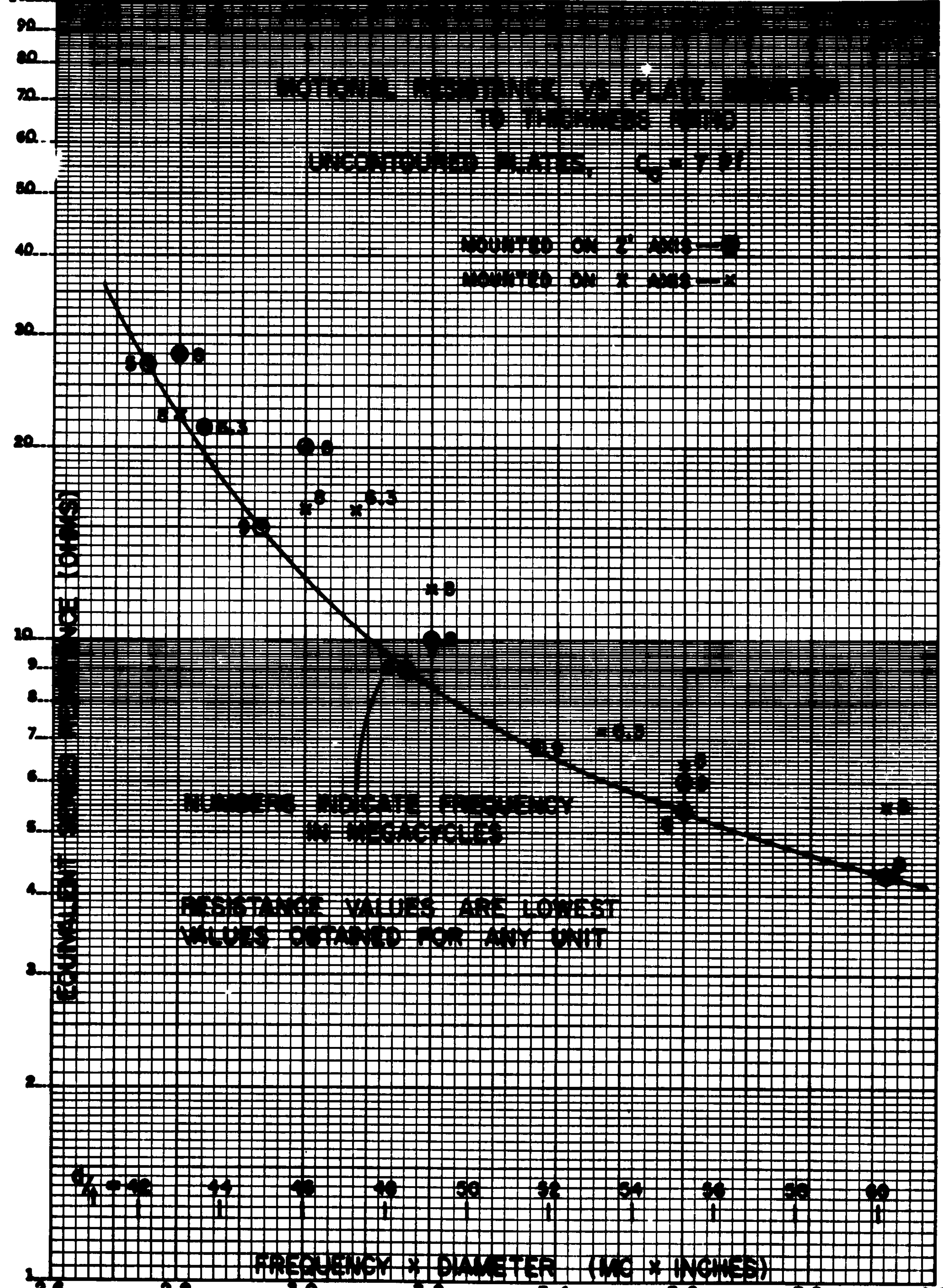
The variation of equivalent resistance with temperature sometimes called "activity dips" is caused primarily by the parasitic modes, as discussed previously. The magnitude of this variation depends on many factors including the types and overtone orders of the parasitic modes encountered as well as the physical parameters which determine the basic motional resistance. In Figures 10, 11, and 12 the minimum and maximum values of the equivalent resistance within the temperature range -55° to $+90^{\circ}C$ are shown for uncontoured plates of various diameters with shunt capacitances near 7 picofarads. The points indicate the mean values for the minimum and maximum resistance within the temperature range, while the vertical lines indicate the range of values obtained for a given group of units.

A comparison of the resistance values of units mounted on the X and Z' axes in Figures 11 and 12 reveals no consistent advantage of one axis over the other. Apparently, the preferred mounting direction will depend upon the particular combination of plate diameter to thickness ratio and electrode size, but unless a small diameter to thickness ratio is used, random orientation is usually satisfactory.

Shunt Capacitance

The capacitance of a capacitor consisting of circular electrodes deposited on a plane-parallel plate of dielectric material is directly proportional to the square of the electrode diameter and inversely proportional to the plate thickness. Since the frequency-thickness coefficient is nearly constant for uncontoured AT plates of the dimensions normally used, the inter-electrode capacitance should be proportional to the product of the square of the electrode diameter and the fundamental frequency. The total shunt capacitance will be the sum of the inter-electrode capacitance and the holder capacitance. Figure 13 is a graph of this function. In Figure 14 the same data has been used to plot the electrode diameter versus frequency for a constant

FIGURE 9



UNCONTOURED PLATES,
MOUNTED ON Z' AXIS
MOUNTED ON X AXIS — x
NUMBERS INDICATE FREQUENCY
IN MEGACYCLES
RESISTANCE VALUES ARE LOWEST
VALUES OBTAINED FOR ANY UNIT
FREQUENCY × DIAMETER (MC × INCHES)

FIGURE 10

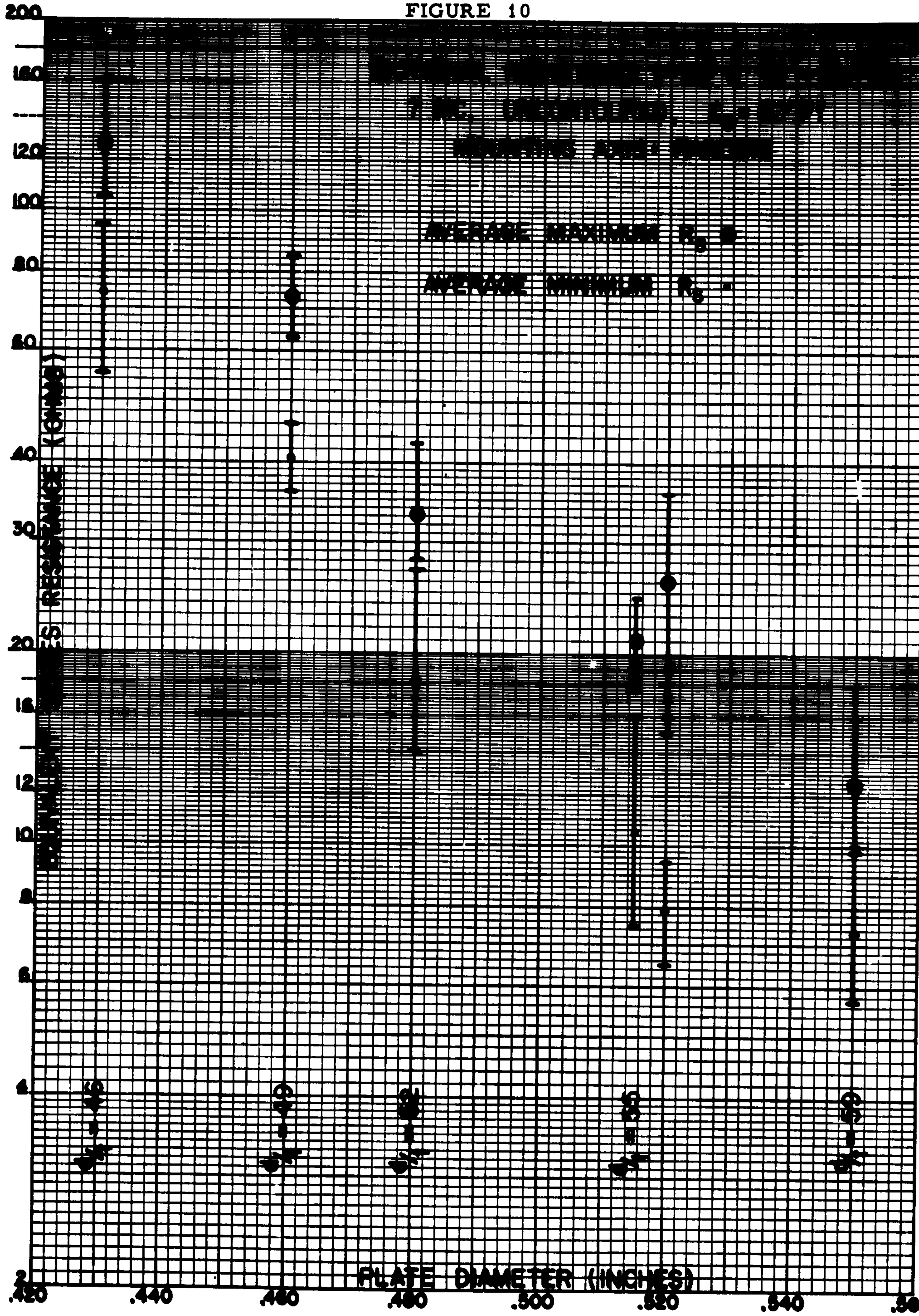

AVERAGE MAXIMUM R_S
AVERAGE MINIMUM R_S
RESISTANCE (OHMS)
PLATE DIAMETER (INCHES)
200
160
120
100
80
60
40
30
20
16
12
10
8
6
4
2
.420
.440
.460
.480
.500
.520
.540
.560

FIGURE 11

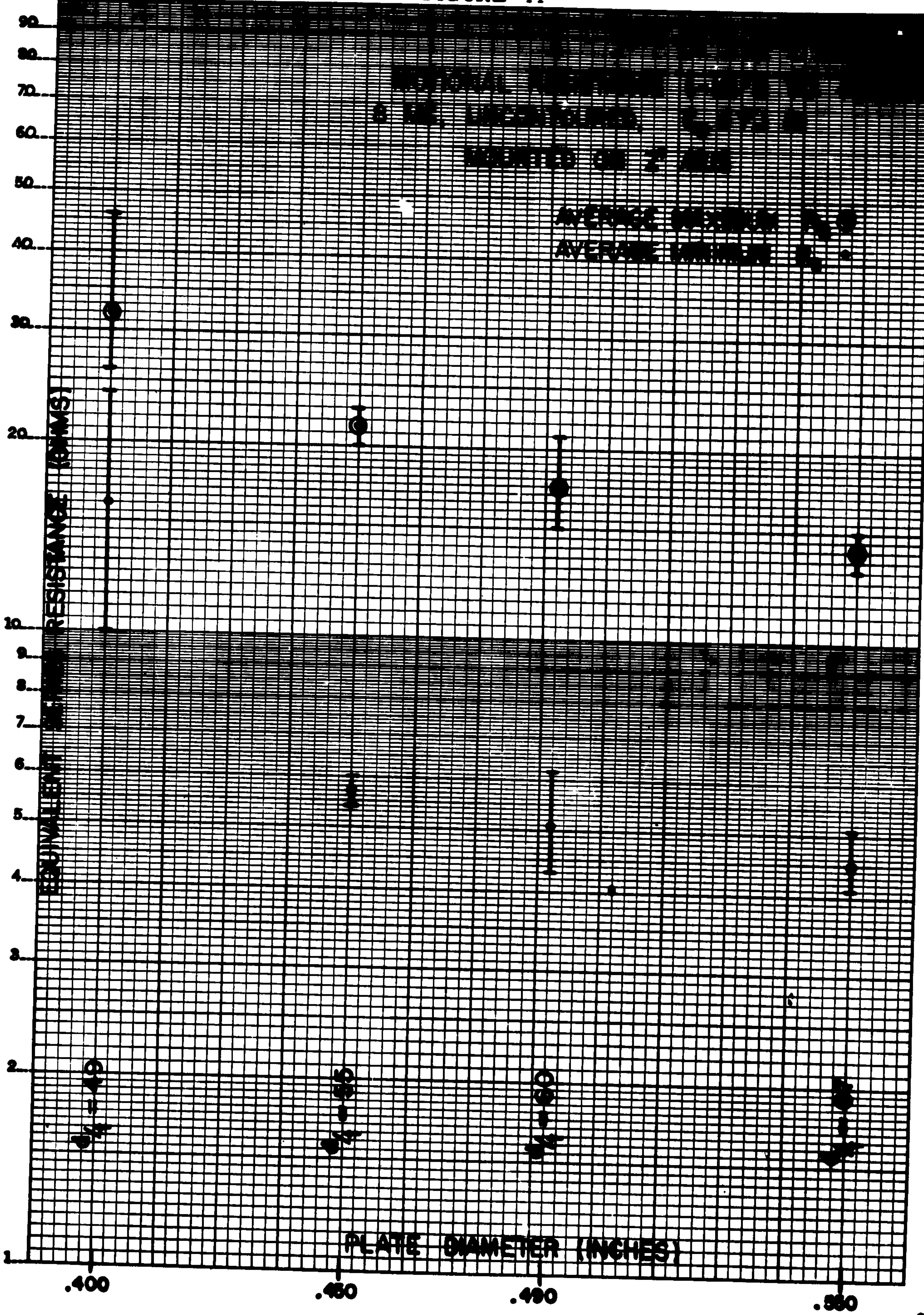
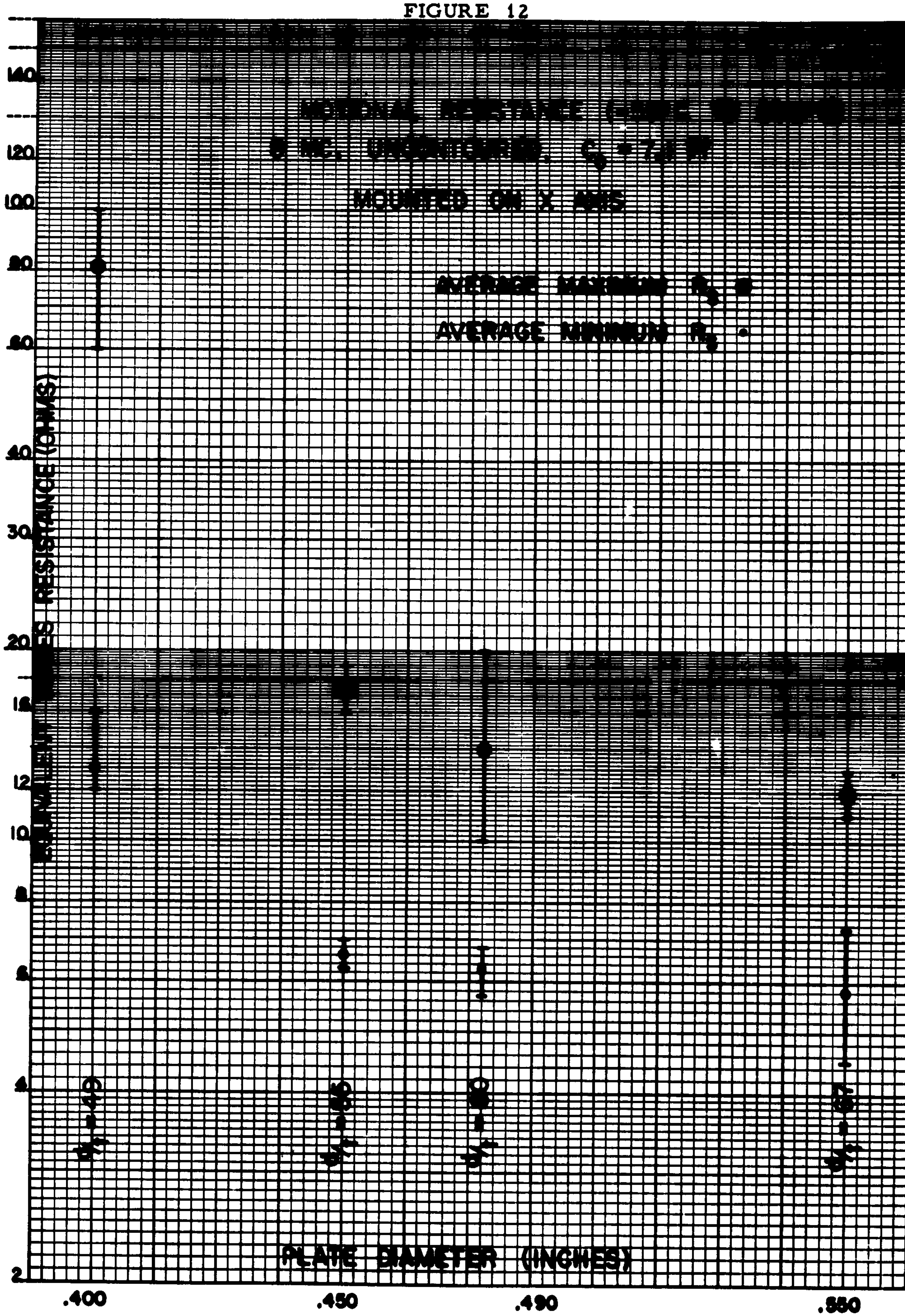

AVERAGE MAXIMUM
AVERAGE MINIMUM
PLATE DIAMETER (INCHES)
.400
.450
.490
.550

FIGURE 12



MOUNTED ON X AXIS
AVERAGE MAXIMUM R
AVERAGE MINIMUM R
RESISTANCE (OHMS)
PLATE DIAMETER (INCHES)
140
120
100
60
40
30
20
10
4
2
.400
.450
.490
.550

FIGURE 13

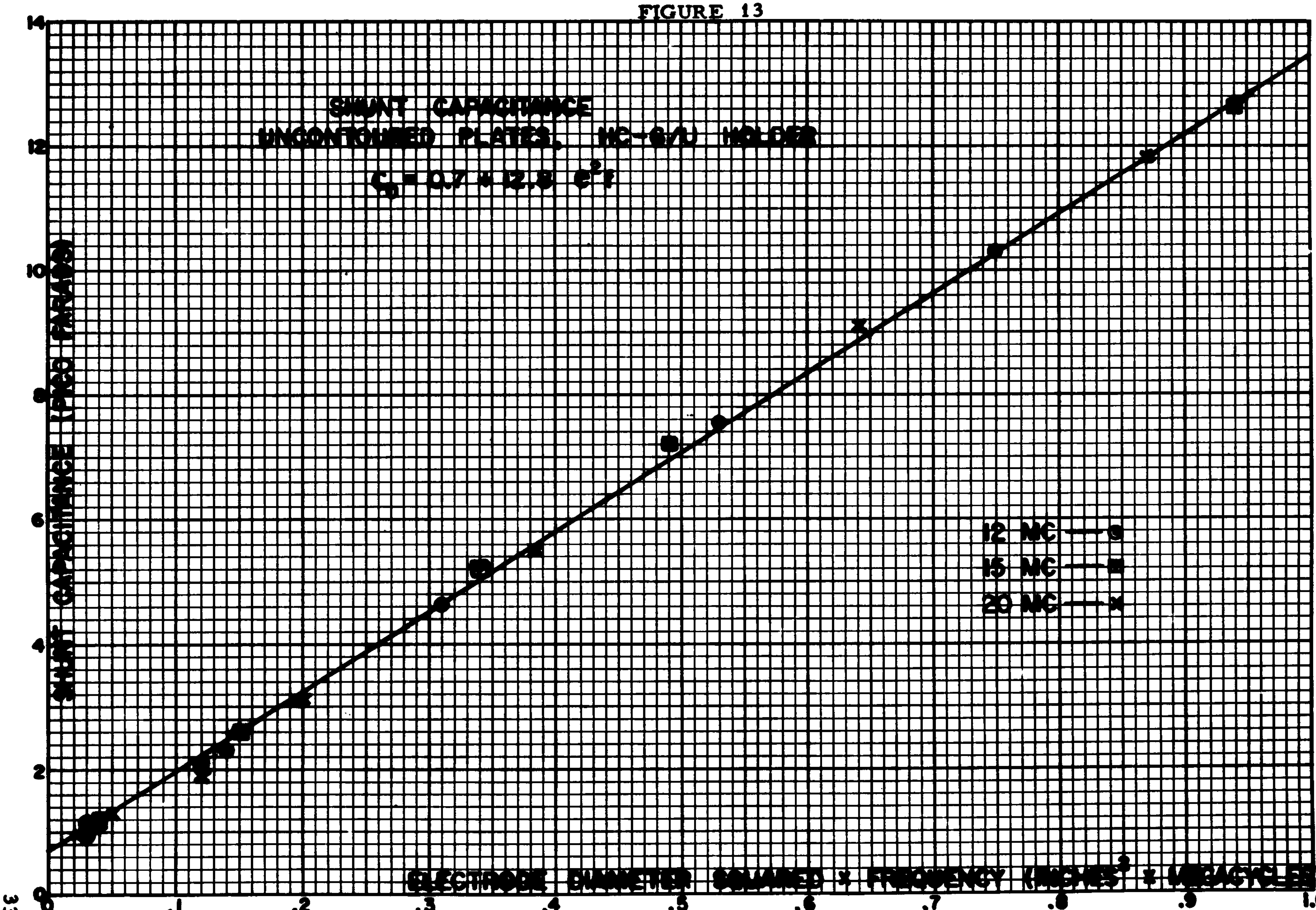
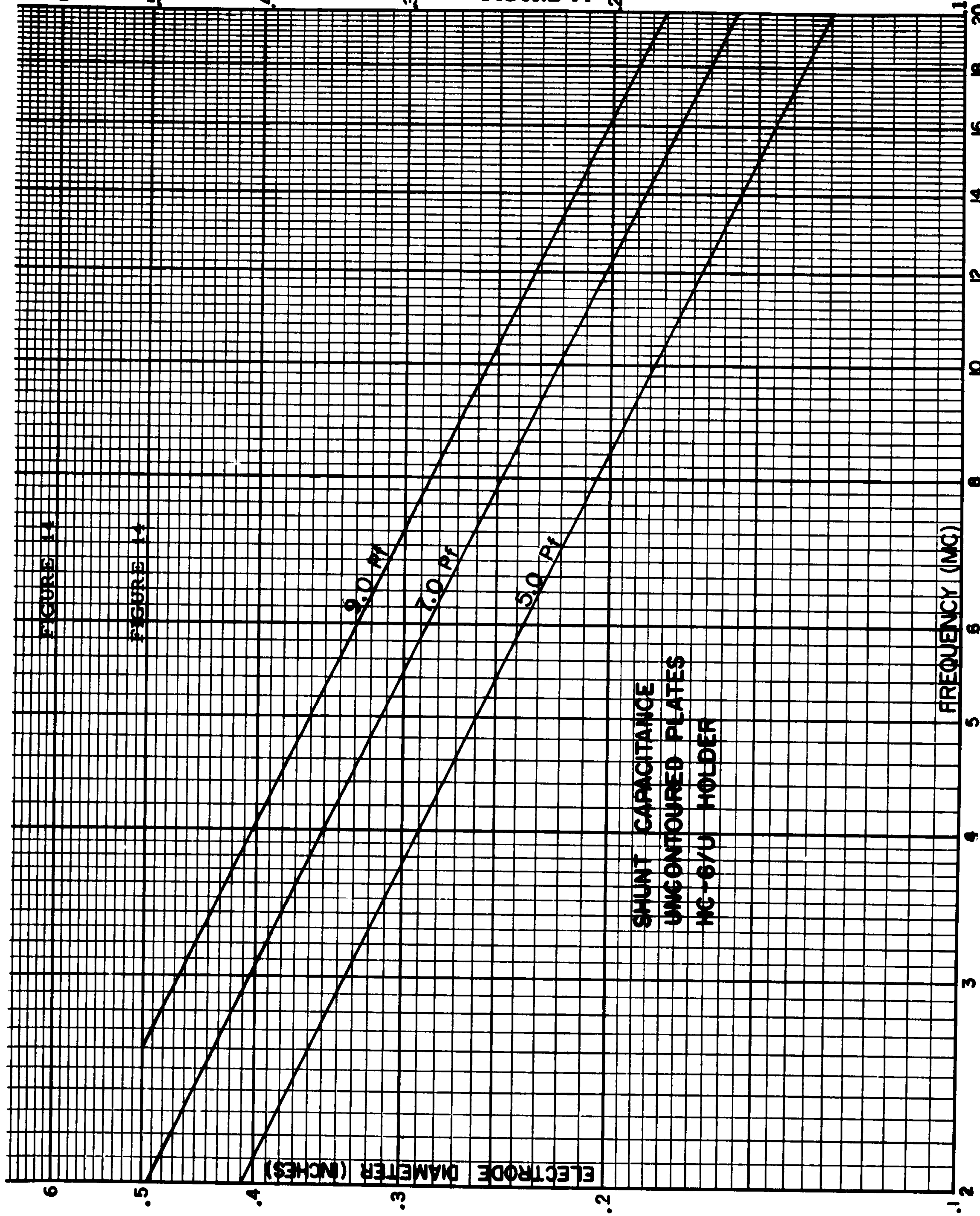


FIGURE 13

FIGURE 14



SHUNT CAPACITANCE
UNCONTOURED PLATES
HC-6/U HOLDER
9.0 Pf
7.0 Pf
5.0 Pf
ELECTRODE DIAMETER (INCHES)
FREQUENCY (MC)
.6
.5
.4
.3
.2
.1
2
3
4
5
6
8
10
12
14
16
18
20
FIGURE 14
FIGURE 14

value of shunt capacitance. In practice, an electrode diameter slightly smaller than that indicated by the graph should be used, to allow for variations in holder capacitance and possible electrode misalignment.

Plating Thickness

The thickness of the metal film comprising the electrode is another factor which affects the equivalent resistance of a resonator. An electrode that is too thin will have an excessive electrical resistance while one that is too thick will have a damping effect on the vibration of the resonator. In either case, the result is an increase in the equivalent motional resistance. Since it is not convenient to control the film thickness by direct measurement, an indirect technique is employed. The application of a metal film to a quartz plate lowers the resonant frequency of the plate by virtue of increasing its mass. Since the decrease in frequency is very nearly proportional to amount of metal applied, the change in resonant frequency affords a convenient method of determining relative electrode thickness.

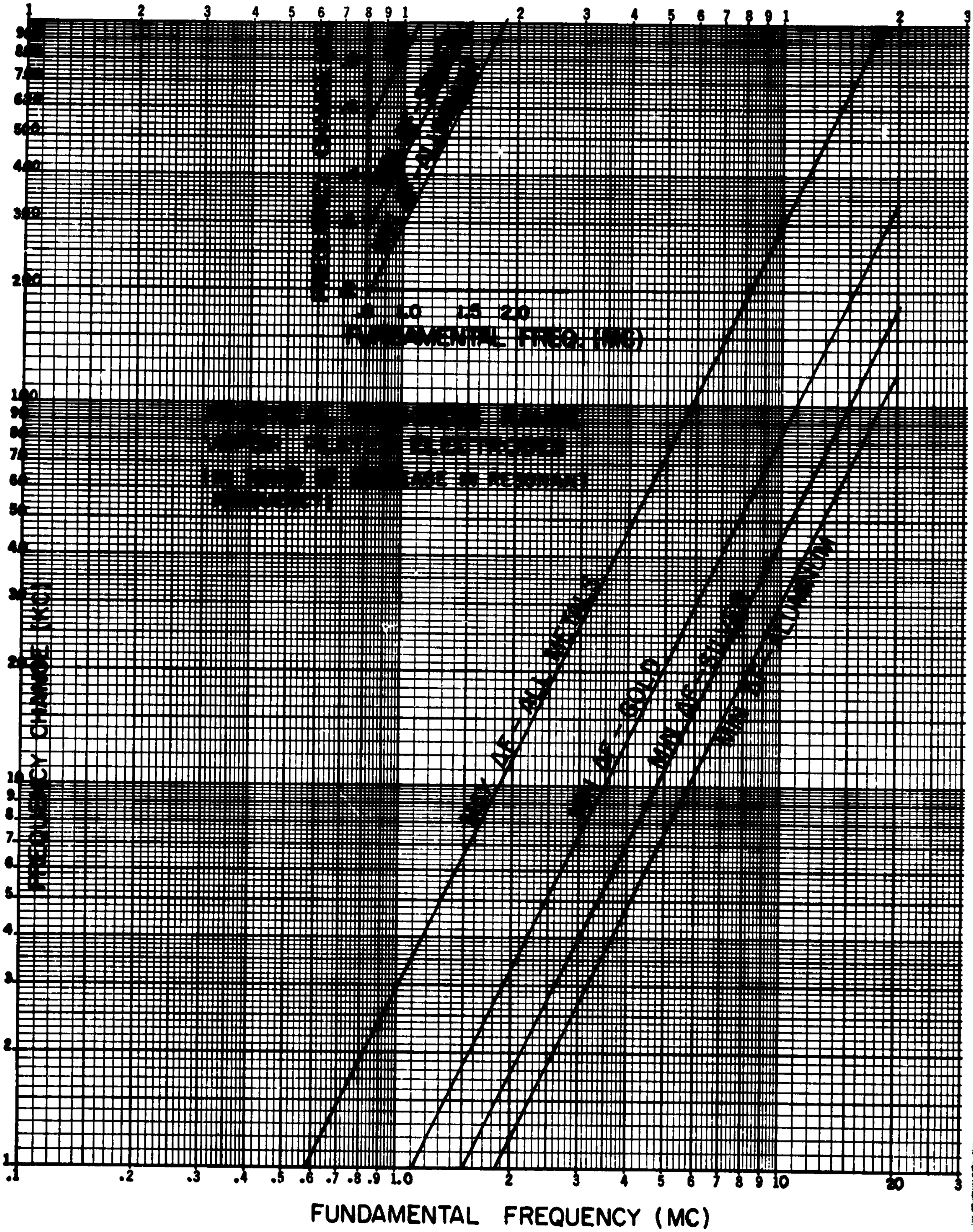
The electrode thickness that will give the most satisfactory results depends upon the metal that is used, the differences being due to the different values of electrical resistivity, density, and internal friction of films of the various metals. Figure 15 is a graph of the minimum and maximum practical plating thickness for gold, silver, and aluminum, in terms of the change in the resonant frequency of the plate. At a frequency of 10 mc, for example, the resonant frequency should decrease during the plating operation by not less than 30 KC nor more than 300 KC if aluminum is used for the electrode material. To minimize frequency aging, a value near the minimum should be used.

B. Plano-Convex Plates

Frequency-Temperature Characteristics

The frequency-temperature curves for fully contoured plates are essentially identical to those obtained with uncontoured plates (see Figure 7), both in respect to the shape of the curves and the

FIGURE 15



angular differences between the "low," "optimum," and "high" angles. There is, however, a significant difference in the actual ZZ' angle required to obtain a given frequency-temperature curve. The apparent effect of making one or both major surfaces spherical is an increase in the ZZ' angle. Consequently, the optimum angle will be lower for a contoured plate than for an uncontoured plate.

The magnitude of this apparent increase in angle is primarily a function of the curvature of the spherical surface. There is some evidence to indicate that the electrode diameter also has an influence, but of a lesser magnitude. Figure 16 is a graph of the apparent change in the ZZ' angle for fully contoured plates as a function of the "relative curvature" (the ratio of the thickness of the quartz plate to the radius of curvature of the spherical surface). Obviously, plates that are geometrically similar as described in section IIC will have the same optimum angle since their relative curvatures are identical. Figure 17 gives the approximate optimum angle for plano-convex plates of various curvatures as a function of the fundamental frequency of the plate.

Motional Resistance

Essentially, two separate effects on the motional resistance of a quartz plate are produced by spherical contouring. The effect on the parasitic modes was discussed in Section IIB. The equivalent resistance of the main mode is also affected by the spherical surface. By reducing the plate thickness at the edge, the vibration is largely confined to the center of the plate. Thus, in effect, the region of maximum vibration is decoupled from the mounting supports. The curvature required to reduce the mounting losses sufficiently, will depend upon the diameter and thickness of the plate, the method of mounting, and the motional resistance specifications which must be met. For the plano-convex resonators covered in this study, the approximate minimum curvature values for various plate diameters are shown in Figure 18.

No general rule for the curvature value which will minimize the influence of the parasitic modes for any plate diameter and frequency can be given, at least at the present time. These mechanically coupled modes are of several varieties: in some cases almost exclusively dependent on the plate diameter, and in other cases they

FIGURE 16

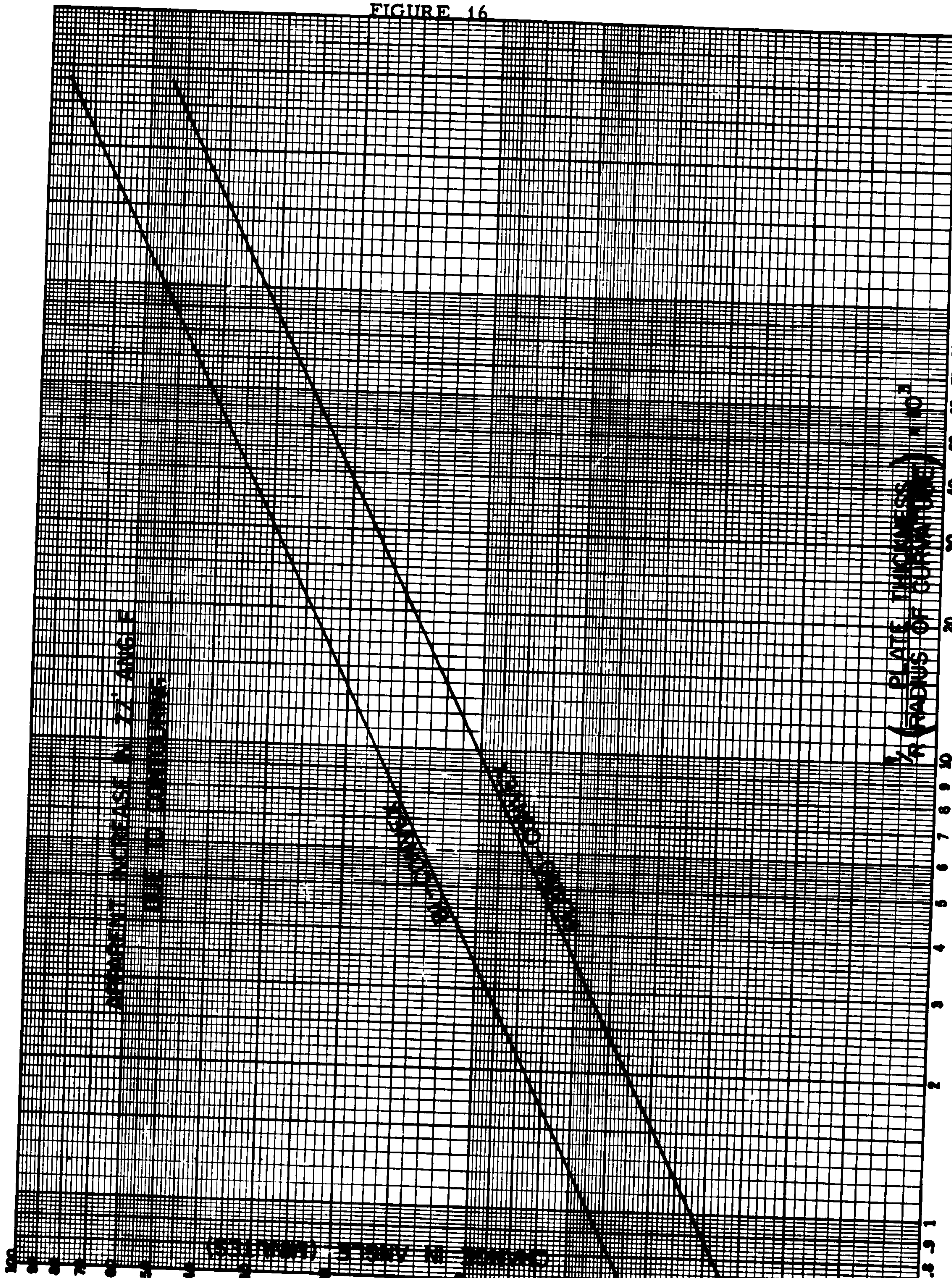

APPARENT INCREASE IN 77° ANGLE
CHANGE IN ANGLE (MINUTES)
t/R (PLATE THICKNESS / RADIUS OF CURVATURE) × 10^3

FIGURE 17

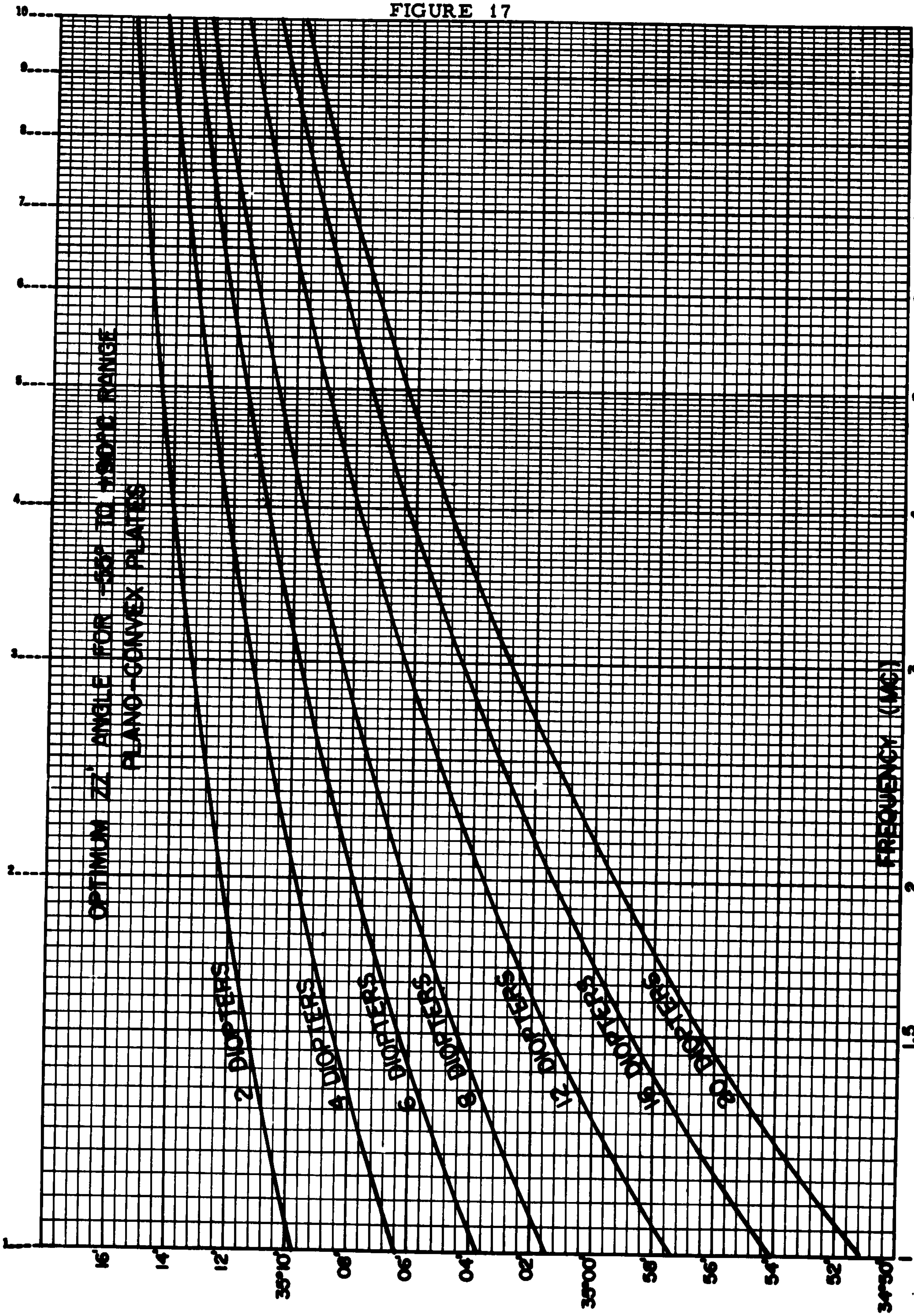
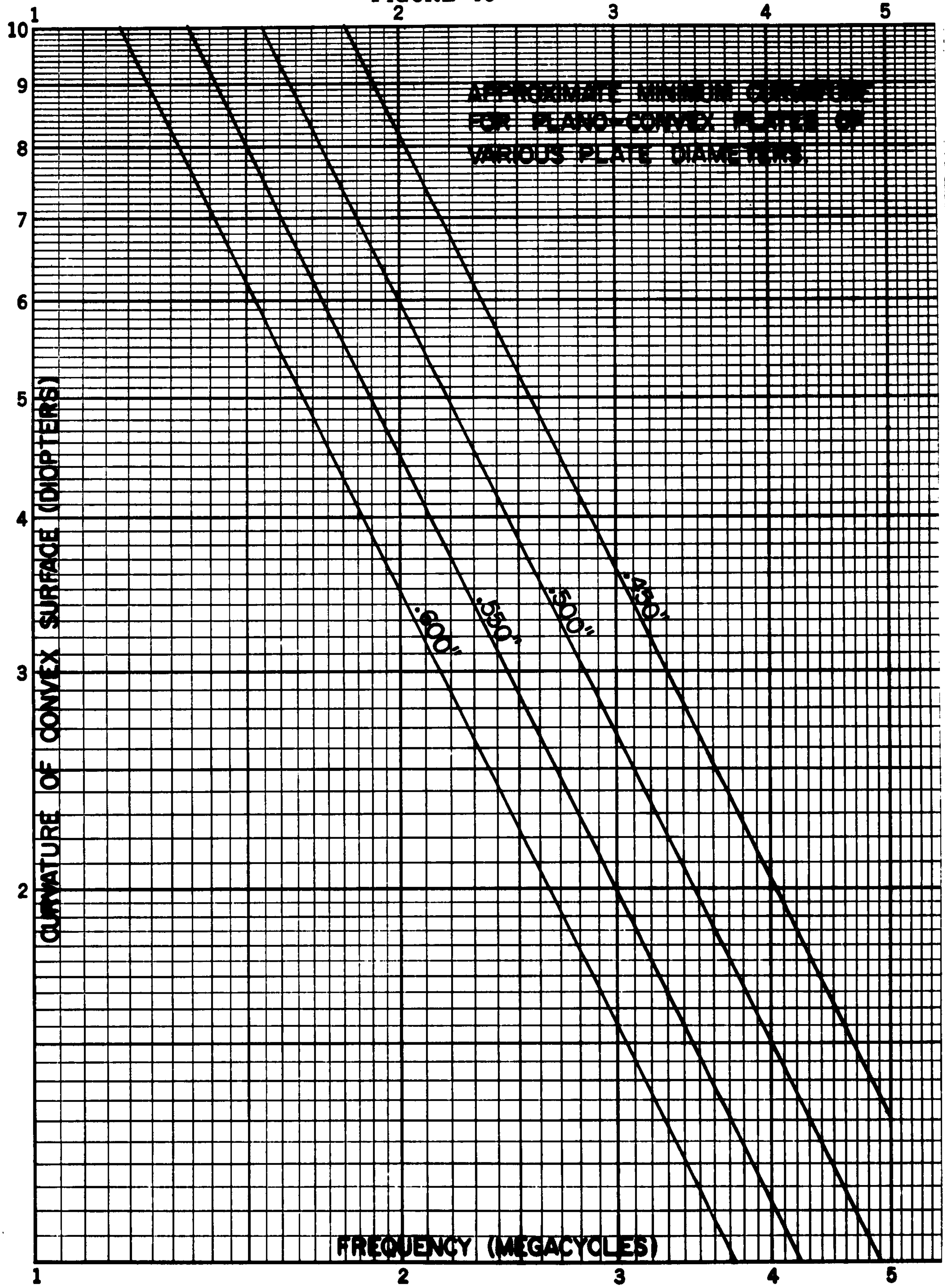

OPTIMUM ZZ' ANGLE FOR -55° TO
RANGE
PLANO-CONVEX PLATES
2 DIOPTERS
4 DIOPTERS
6 DIOPTERS
8 DIOPTERS
12 DIOPTERS
16 DIOPTERS
20 DIOPTERS
FREQUENCY (MC)
1
1.5
2
3
4
5
6
7
8
9
10
16'
14'
12'
35°10'
08'
06'
04'
02'
35°00'
58'
56'
54'
52'
34°50'

FIGURE 18



APPROXIMATE MINIMUM
FOR PLANO-CONVEX
VARIOUS PLATE
CURVATURE OF CONVEX SURFACE (DIOPTERS)
FREQUENCY (MEGACYCLES)
.600"
.550"
.500"
.450"
1
2
3
4
5
6
7
8
9
10

are apparently controlled only by the plate thickness and the curvature. An example of a type of parasitic mode controlled by the plate diameter is shown in Figure 34, Appendix A. This mode couples to the main mode when the frequency-diameter product has certain discrete values, regardless of whether the plate is contoured or plane-parallel.

The effect of the degree of curvature on the motional resistance of a plano-convex plate of relatively large diameter to thickness ratio (in this case about 33) is illustrated in Figure 19. The resistances plotted are the lowest and highest values measured within the temperature range. Each point is the mean value for several resonators having the same curvature. The peak in resistance at 3 diopters is caused by a parasitic mode which is controlled by the plate thickness and curvature rather than the diameter. If the relative curvature is kept constant (the same ratio of plate thickness to radius of curvature) the mode is present regardless of plate diameter.

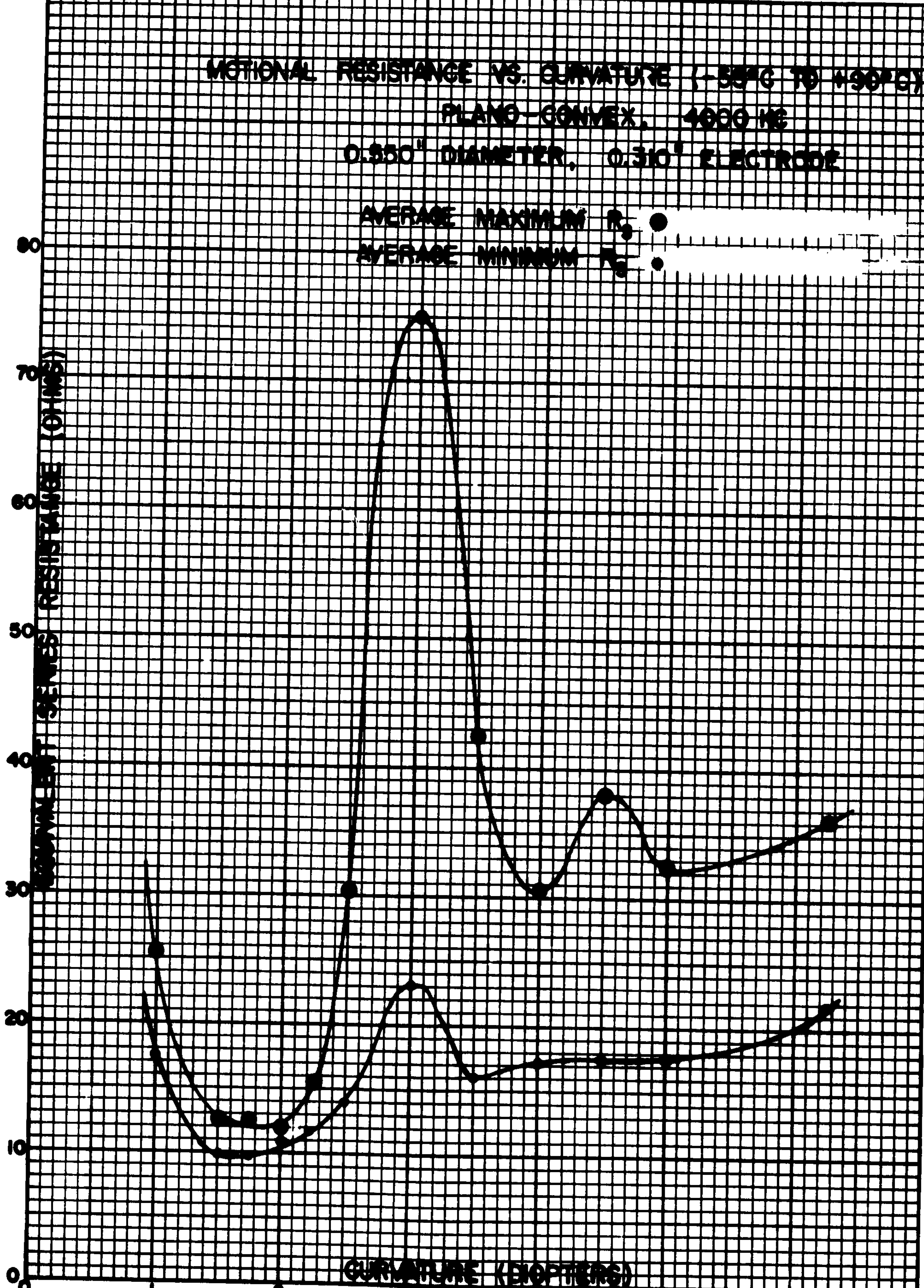
The general trend, as the curvature of a plano-convex resonator is increased, is for the resistance to increase also. This is a basic characteristic, similar to the increase in resistance with decreasing electrode size experienced by uncontoured plates. The increase in curvature can be thought of as reducing the "vibrating area". Strictly speaking, there is some vibration over the whole plate, regardless of curvature, but a greater curvature results in a more rapid decrease in amplitude as we move from the center of the plate toward the edge.

The increase in resistance as the curvature decreases below 1.5 diopters in Figure 19, is due to two factors. Since the plate is becoming more nearly plane parallel, the damping by the mounting supports is increasing. In addition, parasitic modes that are characteristic of uncontoured plates are encountered. The coupling to these modes is reduced sufficiently at the higher curvature values to essentially eliminate their effect.

<u>Optimum Curvature Range</u>

At a curvature of about 2 diopters the variation of resistance over the temperature range is at a minimum. Although the parasitic mode

FIGURE 19


MOTIONAL RESISTANCE VS. CURVATURE (-55°C TO +90°C)
PLANO-CONVEX, 4000 KC
0.550" DIAMETER, 0.310" ELECTRODE
AVERAGE MAXIMUM R_s
AVERAGE MINIMUM R_s
EQUIVALENT SERIES RESISTANCE (OHMS)
CURVATURE (DIOPTERS)
0
10
20
30
40
50
60
70
80
1
2
3
4
5
6
7

causing the peak at 3 diopters is present over a considerable range in curvature, the coupling between it and the thickness-shear mode appears to have a minimum value near 2 diopters. From the Principle of Similarity of Section II, we would expect a similar freedom from parasitic modes at 6 MC if we use a 3 diopter contour or at 8 MC with a 4 diopter contour. This has, in fact, been verified at these and other frequencies. The value of this "optimum" curvature as a function of frequency is shown in Figure 22. The "minimum" and "maximum" curvature lines are intended to indicate a practical range of curvature values over which the equivalent resistance specifications can be met.

If the curvature of a plano-convex plate is near the optimum value, the equivalent resistance of the resonator will be essentially constant over the -55° to +90°C range. However, the value of this resistance will depend on other factors such as the electrode diameter and plate diameter. The effect of varying the electrode diameter is illustrated in Figure 20. When this curve is compared with Figure 8 for uncontoured plates, it is obvious that the effect of electrode size is similar for the two designs, but the magnitude of the effect is much less for contoured plates.

While the electrode diameter is less critical for contoured plates than for uncontoured plates, the plate diameter is, in a sense, even more critical. This is shown in Figure 21. Above a diameter of .400 inch, the equivalent resistance of a 5 MC optimum curvature plate is essentially constant, but as the diameter is decreased below .360 inch, the resistance increases sharply. This point at which the resistance of optimum curvature, plano-convex resonators, begins to increase rapidly is shown as a function of frequency on Figure 22, labeled "minimum plate diameter".

The line labeled "maximum electrode diameter" is the value at which the ratio of the electrode diameter to the plate thickness is about 20. It has been found that under certain circumstances, plano-convex resonators will shift resonant frequency from that of fundamental thickness-shear mode, to that of one of the inharmonic overtones. This, of course, is a function of the selectivity of the oscillator circuit in which the resonator is used, but it is more likely to occur if the electrode diameter is large. With an electrode diameter of no greater than 20 times the plate thickness, this difficulty has not been experienced,

FIGURE 20

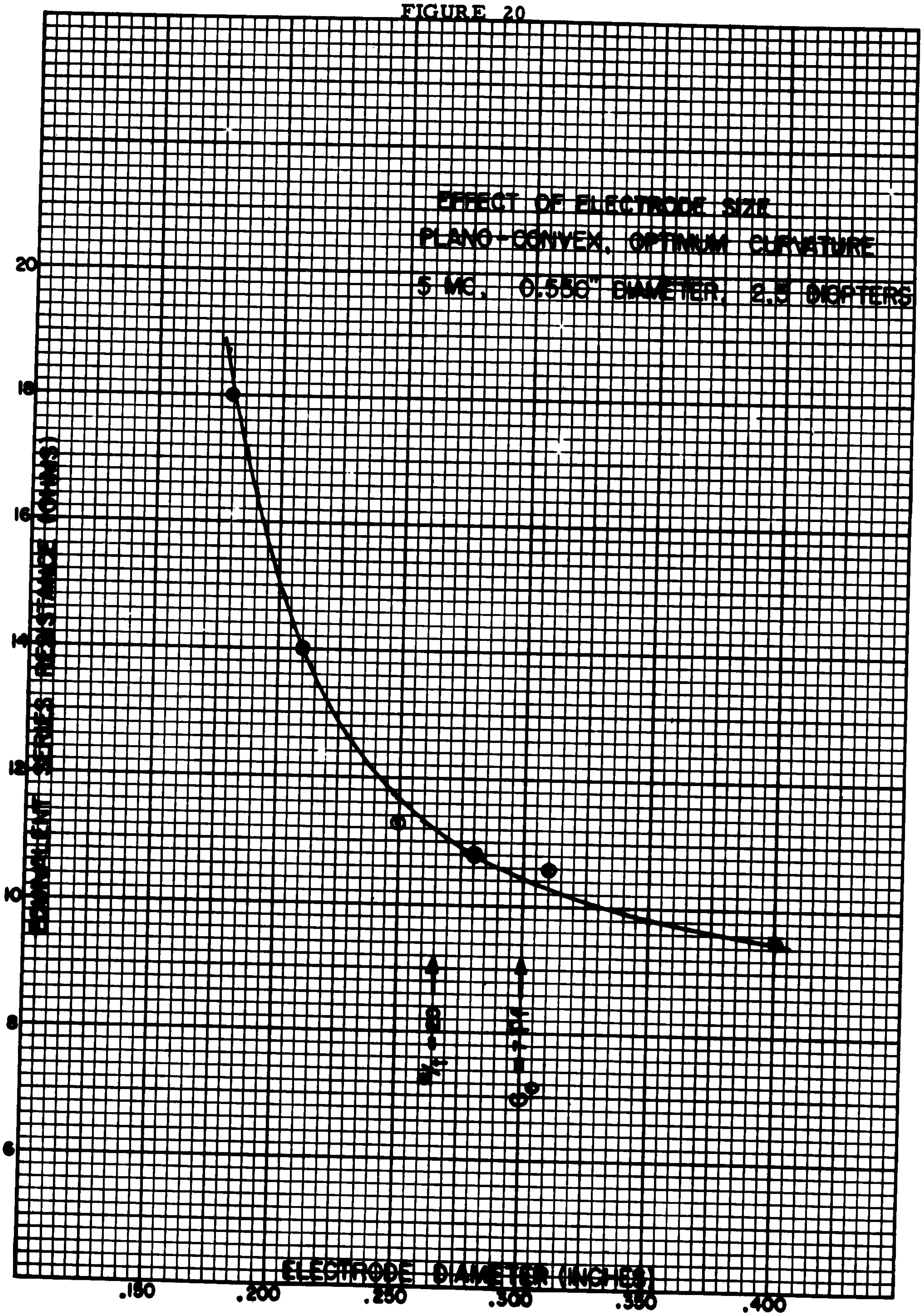

EFFECT OF ELECTRODE SIZE
PLANO-CONVEX, OPTIMUM CURVATURE
5 MC, 0.550" DIAMETER, 2.5 DIOPTERS
EQUIVALENT SERIES RESISTANCE (OHMS)
20
18
16
14
12
10
8
6
ELECTRODE DIAMETER (INCHES)
.150
.200
.250
.300
.350
.400

FIGURE 21

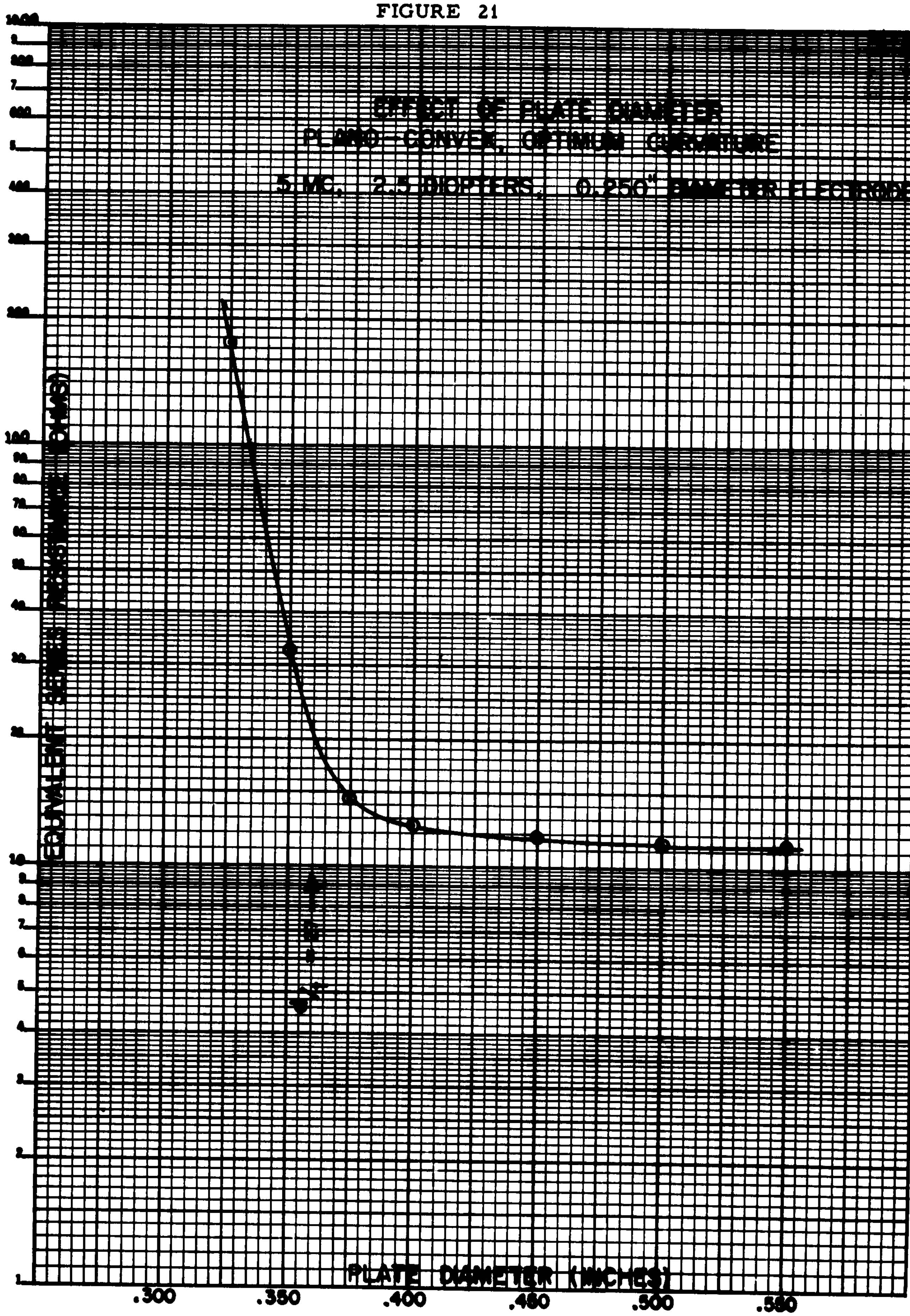
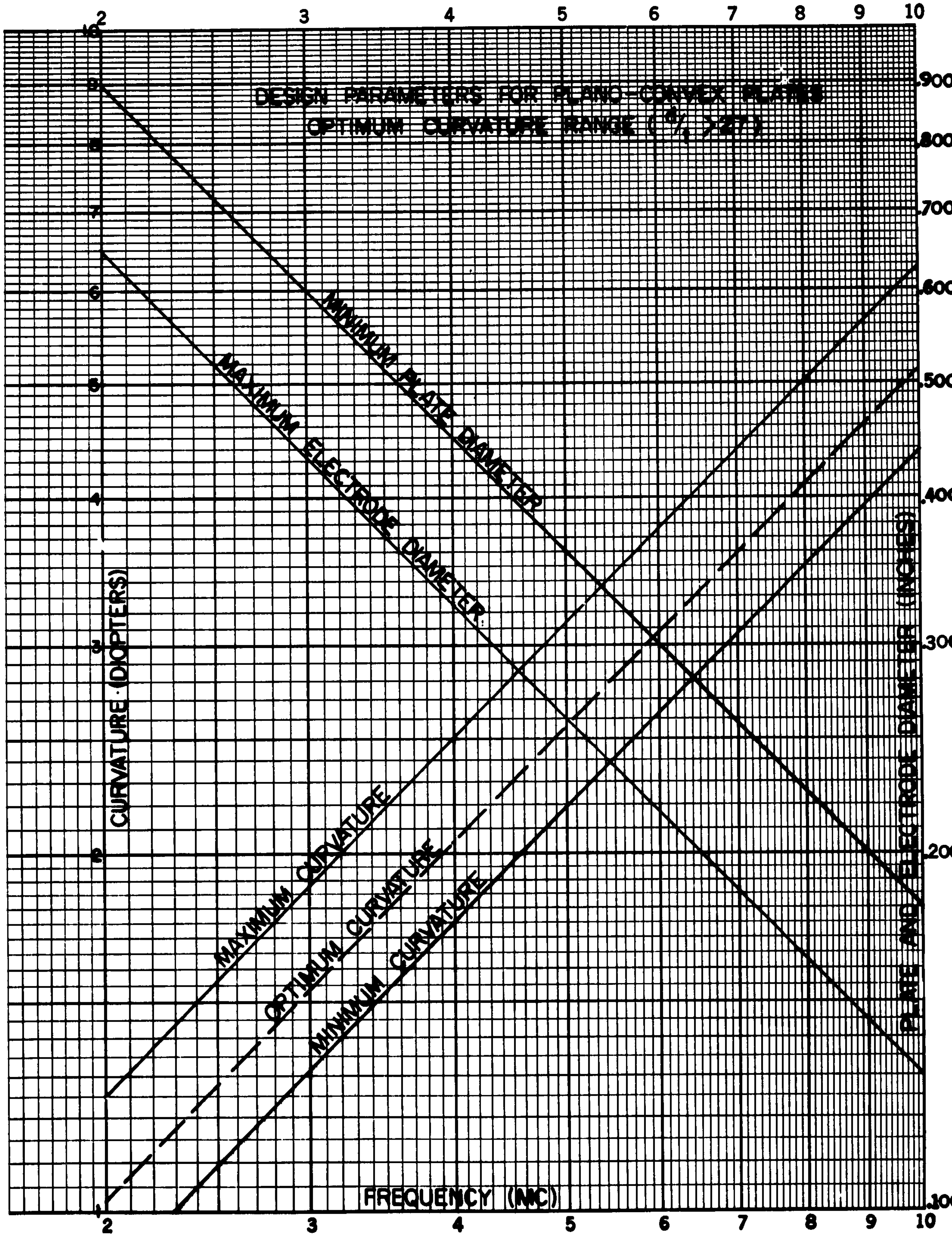

EFFECT OF PLATE DIAMETER
PLANO-CONVEX, OPTIMUM CURVATURE
5 MC, 2.5 DIOPTERS, 0.250" DIAMETER ELECTRODE
EQUIVALENT SERIES RESISTANCE (OHMS)
PLATE DIAMETER (INCHES)
.300
.350
.400
.450
.500
.550
1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000

FIGURE 22



DESIGN PARAMETERS FOR PLANO-CONVEX PLATES
OPTIMUM CURVATURE RANGE ($^{d}/_{t}$ >27)
MAXIMUM PLATE DIAMETER
MAXIMUM ELECTRODE DIAMETER
MAXIMUM CURVATURE
OPTIMUM CURVATURE
MINIMUM CURVATURE
CURVATURE (DIOPTERS)
PLATE AND ELECTRODE DIAMETER (INCHES)
FREQUENCY (MC)
2
3
4
5
6
7
8
9
10
.100
.200
.300
.400
.500
.600
.700
.800
.900

at least with the TS-330/TSM test set.

Intermediate Range

The line labeled "minimum plate diameter" on Figure 22 corresponds to a diameter to thickness ratio of about 27. Below this ratio, plano-convex plates which have the "optimum" curvature are likely to experience excessive damping by the mounting supports. If the dimensions of the enclosure make it impossible to use a plate with a diameter to thickness ratio as large as 27, a higher curvature than the "optimum" value will be necessary. The effect of various values of curvature on the motional resistance of plano-convex plates with diameter to thickness ratios of less than 27 is illustrated by Figures 23, 24, 25 and 26, with diameter to thickness ratios of approximately 24, 20, 18, and 16 respectively. In the first three of these figures, the resistance is high at both ends of the curvature range. At some curvature between the two extremes, the resistance decreases to a minimum. The curvature at which this minimum resistance occurs depends upon the diameter and thickness of the plate. As the diameter to thickness ratio decreases, the range of curvature values over which the resistance is near the minimum value also decreases. At a ratio of 16 (Figure 26) there is no clearly defined minimum in the resistance versus curvature graph.

Within the range of diameter to thickness ratios where the motional resistance versus curvature forms a relatively smooth U-shaped curve, the diameter and curvature of the plate can be regarded as not "critical". That is to say, we would not expect slight changes in the curvature or plate diameter to result in large changes in the motional resistance. This is assuming, of course, that we start with a curvature near the value for minimum resistance. This "non-critical" range of diameter to thickness ratios extends down to a ratio of about 17. The corresponding plate diameters are shown as a function of frequency in Figure 27 labeled "intermediate range".

Critical Range

The general relationship of the plate dimensions to the influence of parasitic modes in a spherically contoured plate can be summarized as follows: a decrease in the diameter to thickness ratio or an increase

FIGURE 23

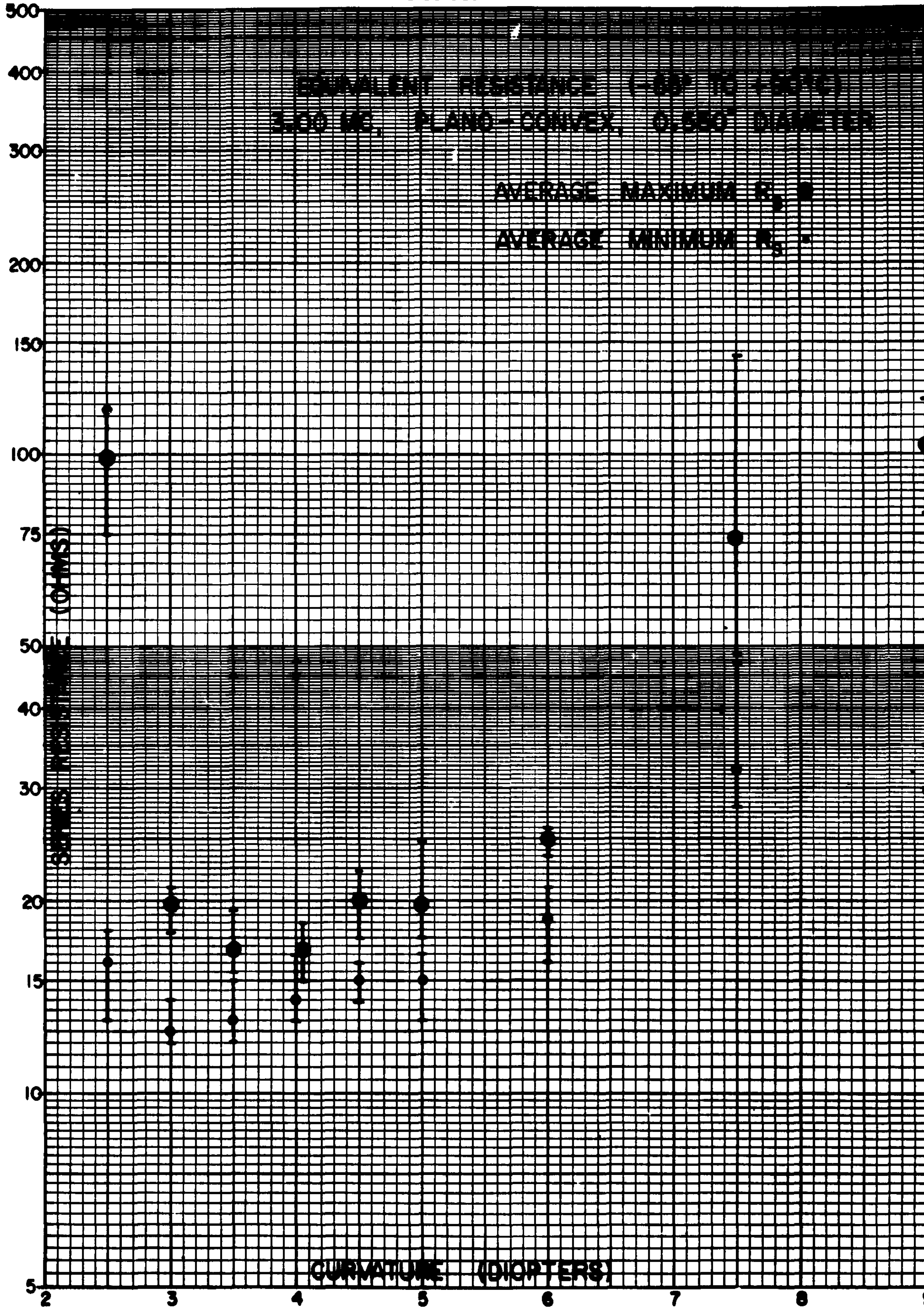
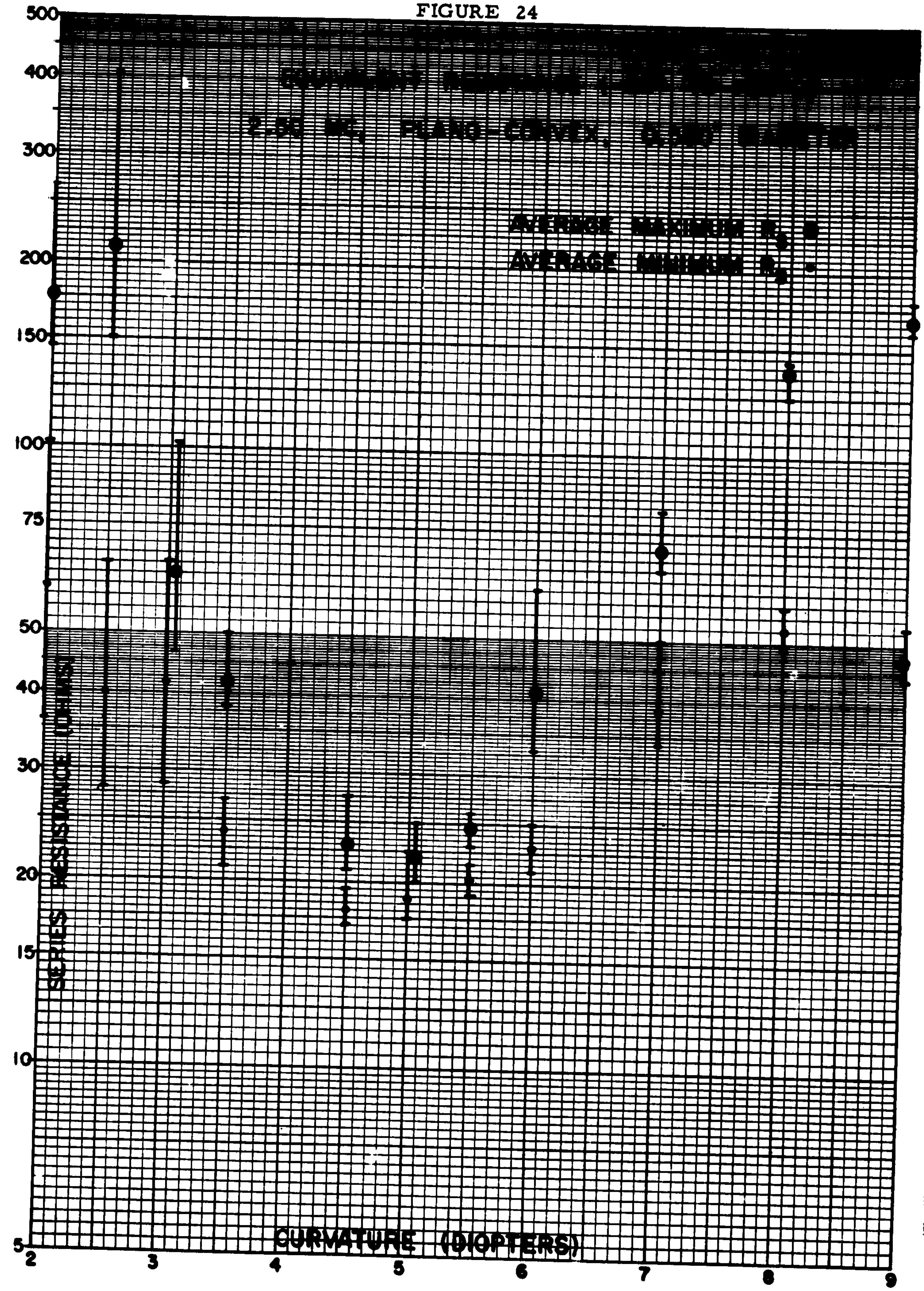
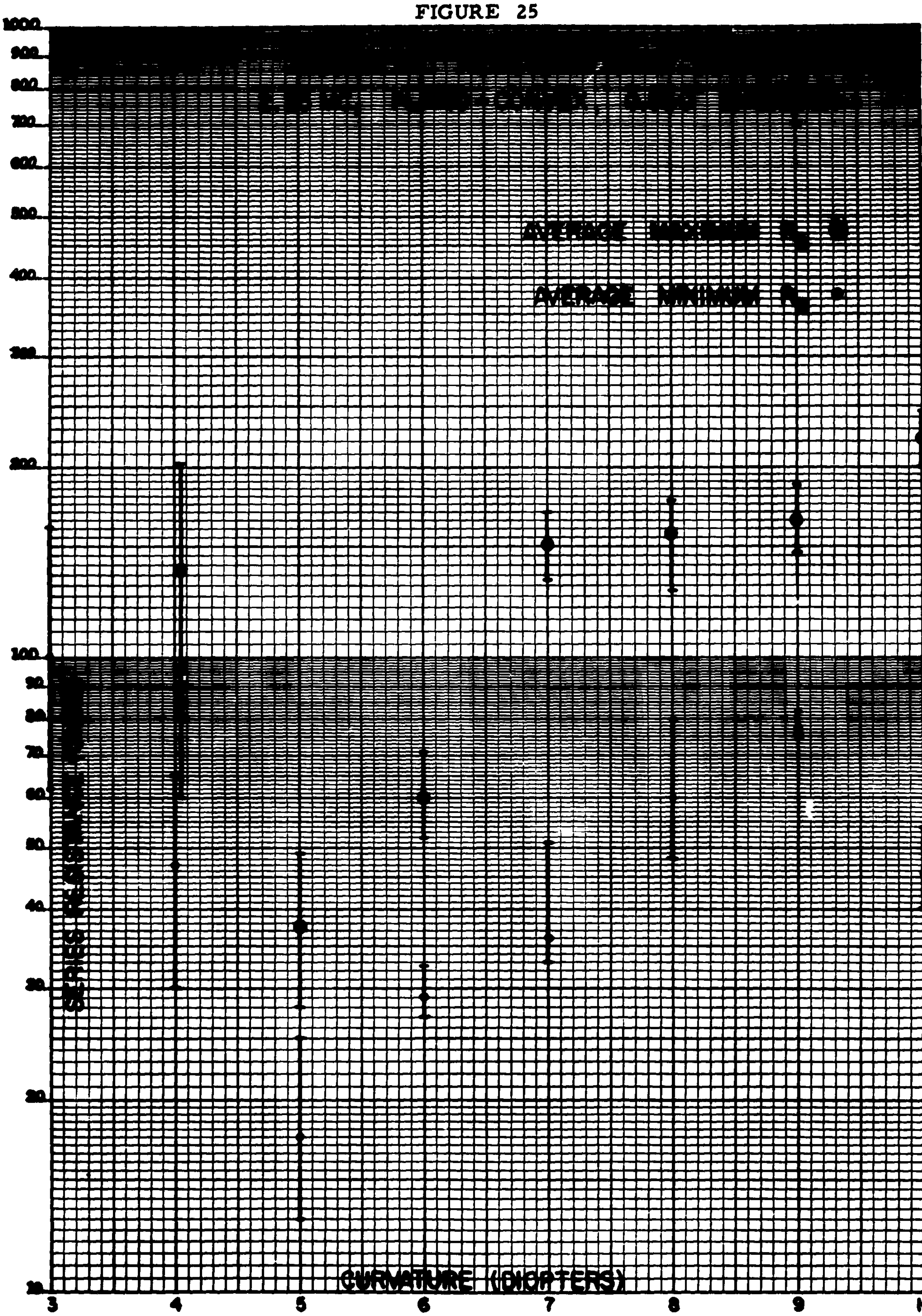

EQUIVALENT RESISTANCE (-65° TO +90°C)
3.00 MC, PLANO-CONVEX, 0.550" DIAMETER
AVERAGE MAXIMUM R_S
AVERAGE MINIMUM R_S
SERIES RESISTANCE (OHMS)
CURVATURE (DIOPTERS)
500
400
300
200
150
100
75
50
40
30
20
15
10
5
2
3
4
5
6
7
8
9

FIGURE 24



AVERAGE MAXIMUM R_s
AVERAGE MINIMUM R_s
SERIES RESISTANCE (OHMS)
500
400
300
200
150
100
75
50
40
30
20
15
10
5
CURVATURE (DIOPTERS)
2
3
4
5
6
7
8
9

FIGURE 25



AVERAGE MAXIMUM R_S
AVERAGE MINIMUM R_S
CURVATURE (DIOPTERS)
SERIES RESISTANCE

FIGURE 26

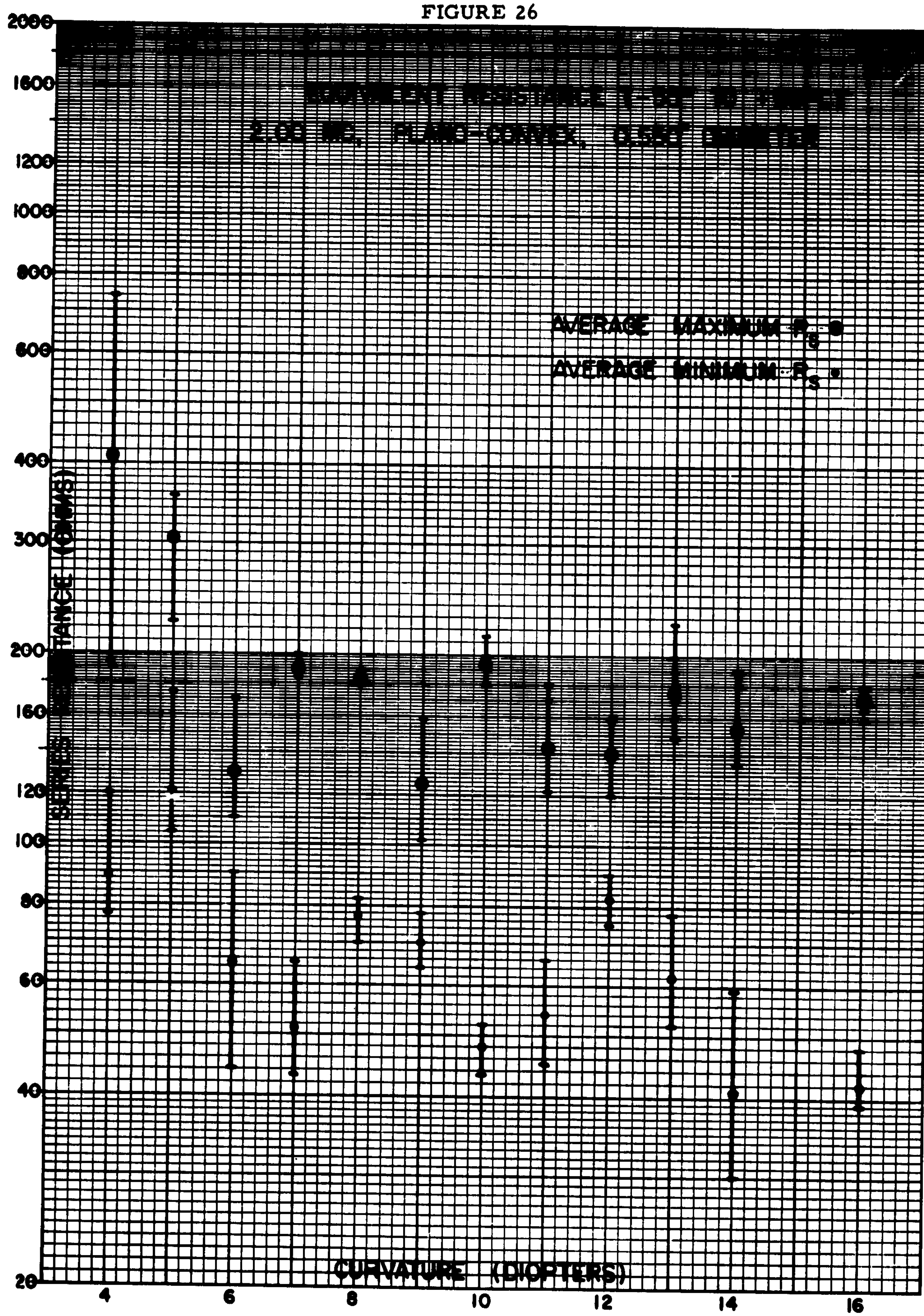
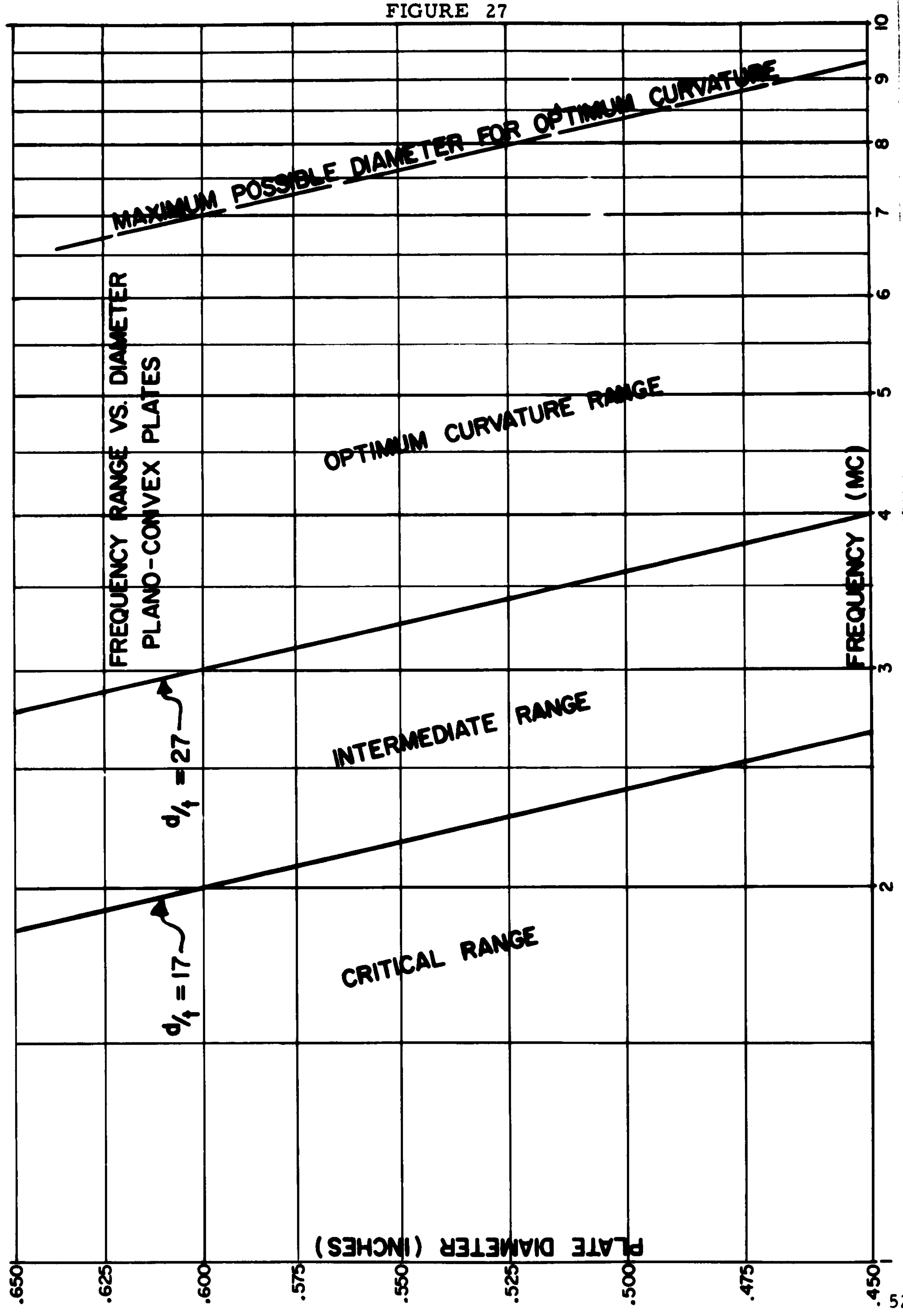

AVERAGE MAXIMUM R_S
AVERAGE MINIMUM R_S
SERIES RESISTANCE (OHMS)
CURVATURE (DIOPTERS)
2000
1600
1200
1000
800
600
400
300
200
160
120
100
80
60
40
20
4
6
8
10
12
14
16

FIGURE 27



FREQUENCY RANGE VS. DIAMETER
PLANO-CONVEX PLATES
MAXIMUM POSSIBLE DIAMETER FOR OPTIMUM CURVATURE
OPTIMUM CURVATURE RANGE
INTERMEDIATE RANGE
CRITICAL RANGE
$d/_t = 27$
$d/_t = 17$
FREQUENCY (MC)
10
9
8
7
6
5
4
3
2
PLATE DIAMETER (INCHES)
.650
.625
.600
.575
.550
.525
.500
.475
.450

in the relative curvature will increase the effect of the parasitic modes. Since a small diameter to thickness ratio necessitates a high relative curvature to avoid excessive mounting losses, the problem of avoiding strong parasitic mode coupling is compounded. For plano-convex plates, the diameter to thickness ratio at which the plate parameters become "critical" is about 17. When the ratio is smaller than this, there is no simple rule to follow in choosing plate parameters which will insure freedom from strong parasitic mode effects. The procedure then is to try various combinations of plate diameter and curvature until a suitable combination has been found for the frequency in question. A set of plate parameters for a different frequency can be determined by applying the Similarity Principle or by another trial and error process.

The maximum value of the equivalent resistance obtained within the -55° to $+90^{\circ}$C range is compared to that allowed by MIL-C-3098B specification for the .550" diameter plano-convex resonators fabricated for this study in Figure 28. Each point represents a group of units of a particular frequency and curvature. The symbols indicate whether all, part, or none of the units met the resistance specifications. Those designs which gave the lowest maximum resistance, together with the best results obtained with other plate diameters, have been extrapolated following the Similarity Principle to provide designs covering the frequency range from 1.4 to 2.0 MC which should prove satisfactory. These are shown in Table 1.

C. Bi-Convex Plates

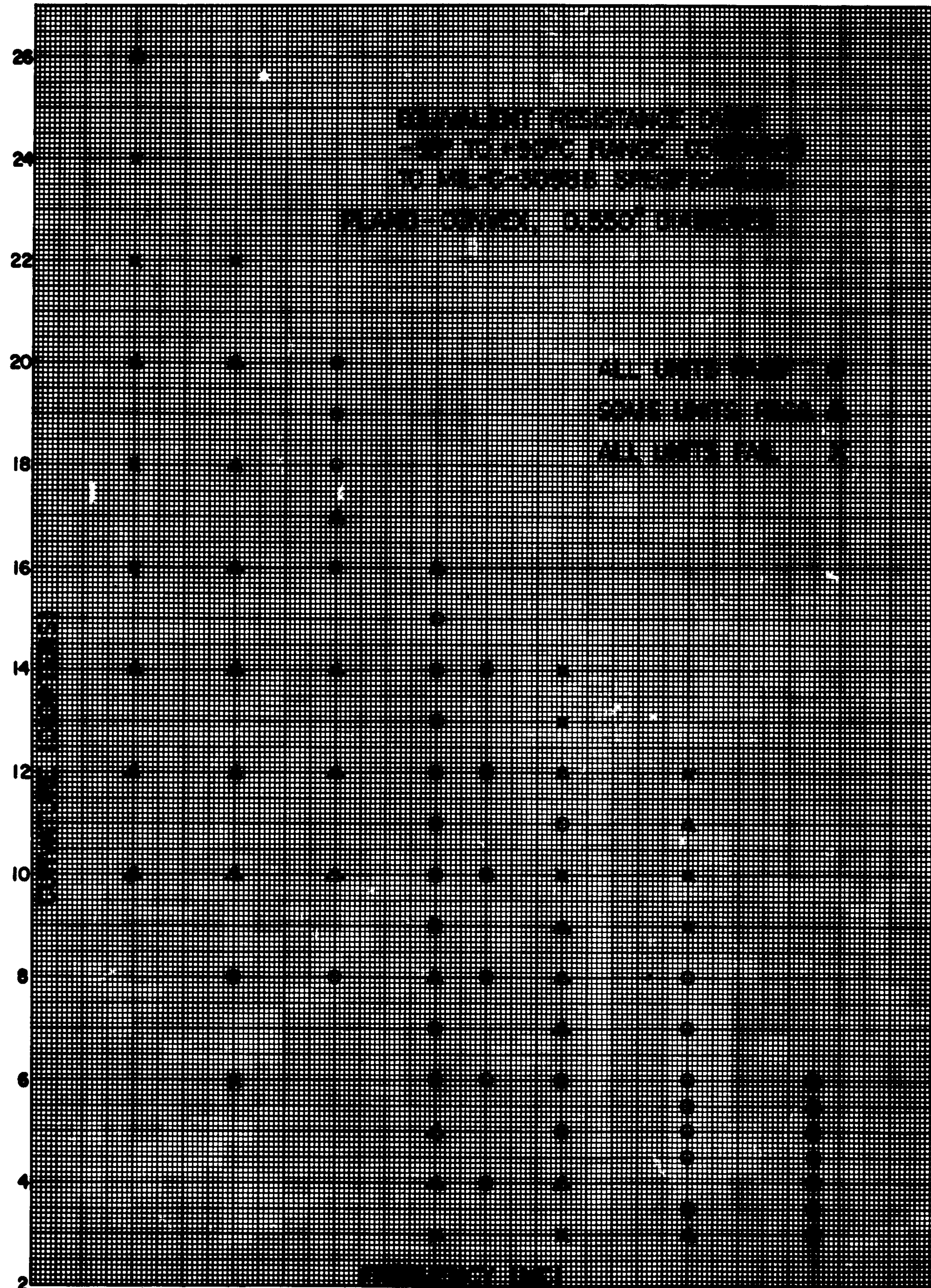
Frequency-Temperature Characteristics

The effect of applying a spherical contour to both major surfaces of an AT plate is similar to that of contouring only one surface but, as might be expected, it is of greater magnitude. The apparent change in the ZZ' angle was discussed in section IIIB and its magnitude as a function of the relative curvature is shown in Figure 16. Figure 29 is a graph of the "optimum" or minimum frequency deviation angle for bi-convex plates of various curvatures.

Motional Resistance

The general statements that were made concerning the effects of the plate dimensions on the motional resistance of plano-convex

FIGURE 28



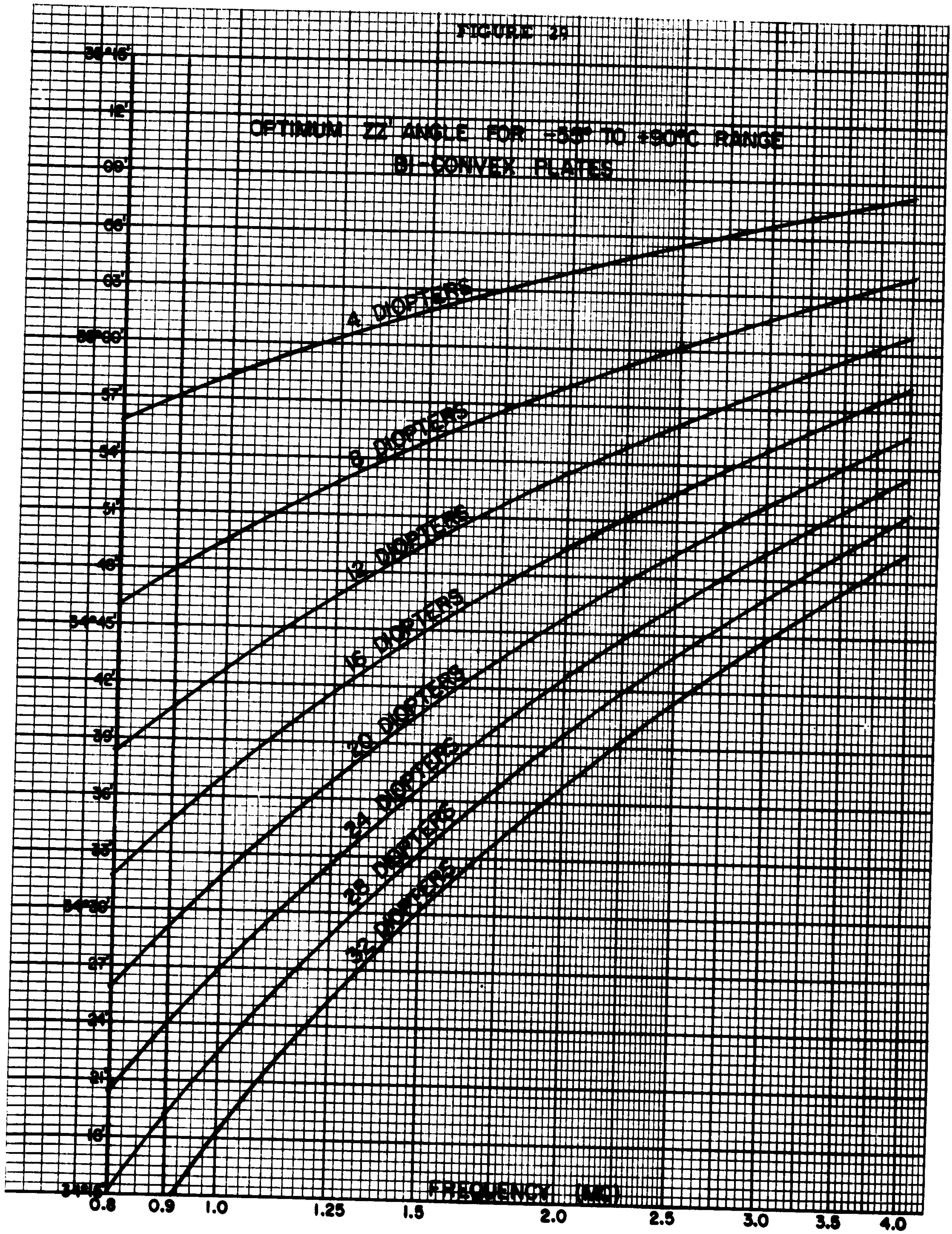


FIGURE 29
OPTIMUM ZZ' ANGLE FOR -55° TO +90°C RANGE
BI-CONVEX PLATES
4 DIOPTERS
8 DIOPTERS
12 DIOPTERS
16 DIOPTERS
20 DIOPTERS
24 DIOPTERS
28 DIOPTERS
32 DIOPTERS
FREQUENCY (MC)
0.8
0.9
1.0
1.25
1.5
2.0
2.5
3.0
3.5
4.0

plates may also be applied to bi-convex plates but the numerical values for such relationships as the minimum curvature versus frequency (Figure 18) will be different. The advantage of the bi-convex design as compared to the plano-convex is that, for a plate of a given diameter to thickness ratio, a lower value of motional resistance can apparently be obtained. From the information presently available, however, this advantage is of significant magnitude only when the ratio is less than about 14. The reason for the superior performance of the bi-convex design is not definitely known, but it may be due to a weaker coupling between the thickness-shear mode and the parasitic modes, stemming from the more balanced distribution of mass about the "nodal plane" through the center of the plate.

The diameter and curvature of a bi-convex plate are both "critical" when the diameter to thickness ratio is small enough to make it advantageous to use this design instead of the plano-convex. A small change in either parameter can cause a considerable change in the performance of the resonator.

The parasitic modes associated with bi-convex plates of small diameter to thickness ratio are dependent upon all of the plate dimensions but, depending upon the mode of vibration involved, some are influenced more by the plate diameter than by the curvature. For other modes the reverse is true.

Various overtone orders of three modes that are primarily dependent on the plate diameter are shown in relation to the thickness-shear mode for circular; plane-parallel, AT-cut plates in Figure 33, Appendix A. These curves were plotted from experimental data obtained in this study and from the literature. Since this data represents plane-parallel plates, we would not expect the points of intersection of the thickness-shear line and the parasitic mode lines to have the same values of d/t and fxt for bi-convex plates. What is needed is a three-dimensional plot of fxt, d/t, and t/R to predict where the parasitic modes will coincide with the thickness-shear in bi-convex plates, but a general idea of the problem can be obtained from this chart.

Figure 30, 31, and 32 compare the maximum equivalent resistance of bi-convex plates with various combinations of plate diameter, thickness, and curvature with the military specifications for resistance. Each point represents a group of units, usually about 5 in number. The

FIGURE 30

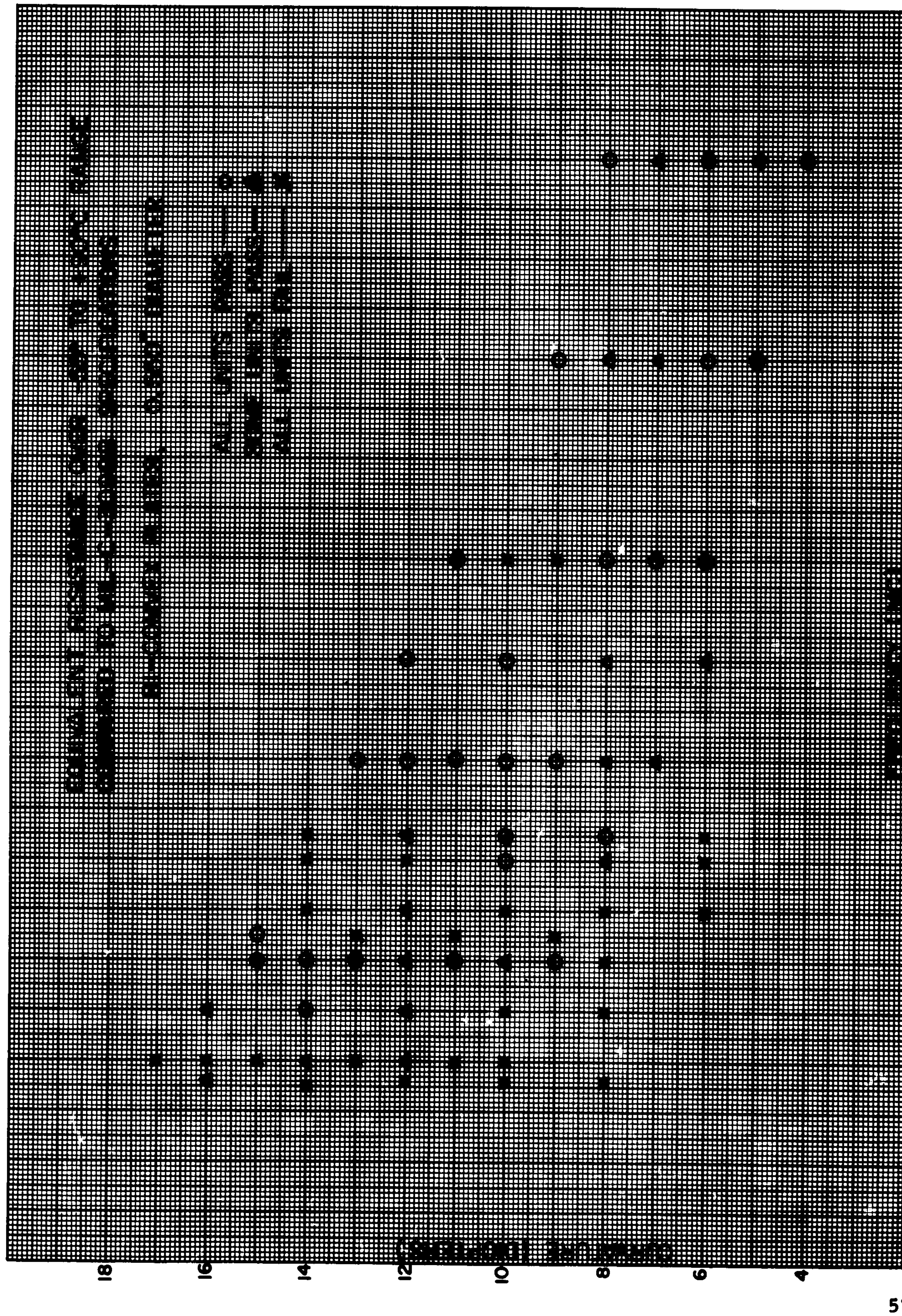
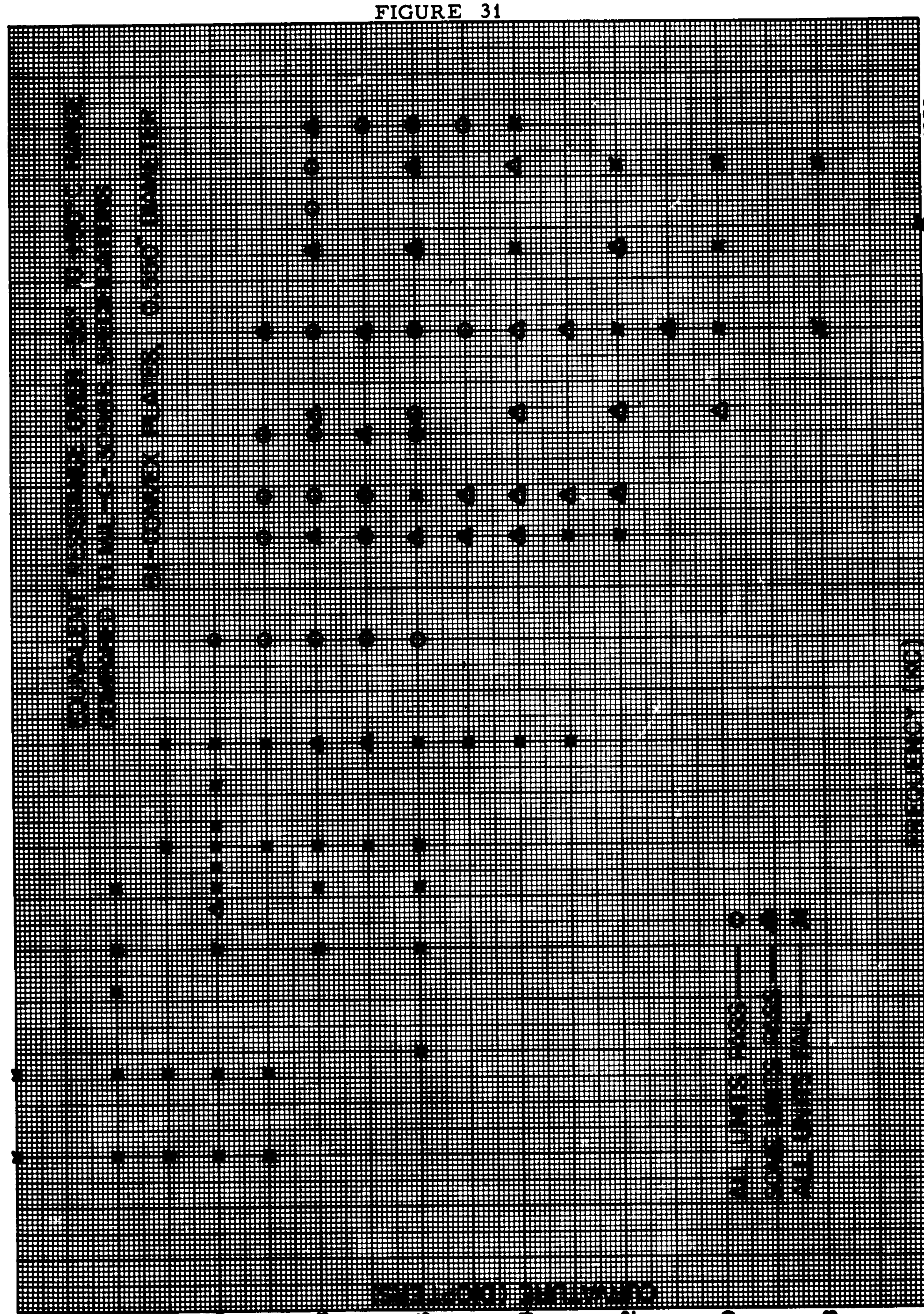
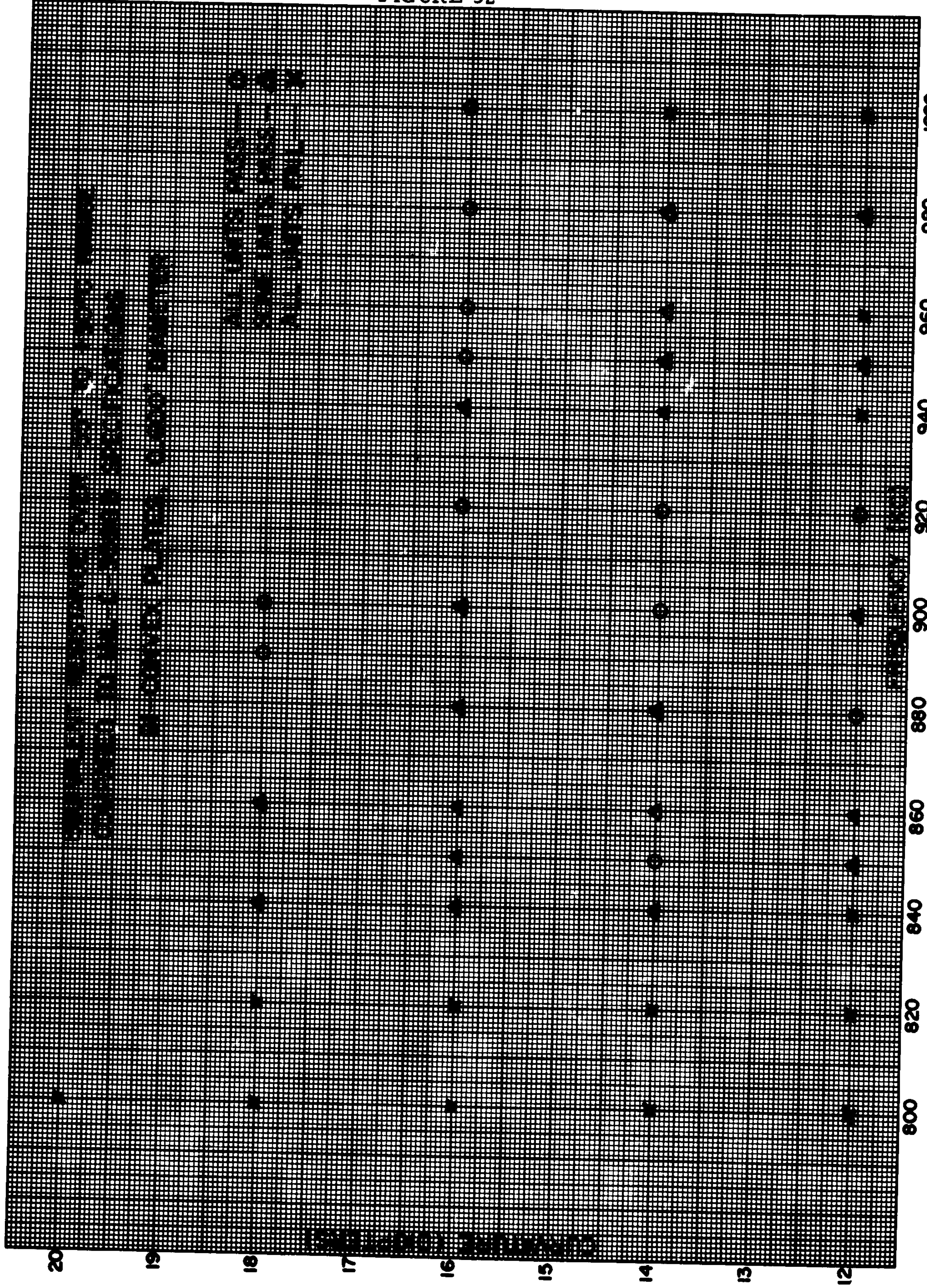


FIGURE 31



BI-CONVEX PLATES, 0.550" DIAMETER
ALL UNITS PASS
SOME UNITS PASS
ALL UNITS FAIL
FREQUENCY (KC)
800
825
850
875
900
925
950
975
1000
1025
1050
6
8
10
12
14
16
18
20
22
24

FIGURE 32



800
820
840
860
880
900
920
940
960
980
1000
12
13
14
15
16
17
18
19
20

actual values of the equivalent resistance for these units as well as those of other plate diameters are listed in Table 5. It is apparent from studying this data, that considerably more information would be required to develop a complete picture of the subject, but a few general conclusions can be made from the available data. A strong interfering mode appears to be present when the frequency is near 1.1 MC with a .550 inch plate. This same mode should occur near 1.0 MC with a .600 inch plate, which would explain why a smaller diameter than .600 gives better results at this frequency. Below 900 kilocycles, all designs with a .550 inch plate proved unsatisfactory. This corresponds in diameter to thickness ratio to about 825 kilocycles for a .600 inch plate, which appears to be near the lower frequency limit for plates of this size. The available data is insufficient to definitely establish that these are the lowest frequencies at which satisfactory results can be obtained with bi-convex plates of these sizes, but that is the indication. If this is true, we would expect that the minimum diameter for an 800 kilocycle plate would be about .620 inch. In terms of diameter to thickness ratio, this lower limit corresponds to a ratio of about 6.5. It is interesting to note that at this ratio, three parasitic modes coincide with the main mode in Figure 33.

The bi-convex designs which gave the best results in terms of maximum equivalent resistance over the temperature range have been extrapolated to other frequencies by means of the Similarity Principle. These calculated designs cover the range 800 kilocycles to 1500 kilocycles and are shown in Table 2. The values for plate diameter and curvature are computed to the nearest mil and tenth diopter, respectively. It should not be necessary, in most cases, to duplicate these values exactly in order to obtain satisfactory performance, but since the available data is not adequate to establish tolerances, none have been given. The designs given are not the only ones that will be sufficiently free of parasitic modes nor, in all probability, the best that can be found.

The extrapolated design parameters of Table 2 are limited to plate diameters less than .650 inch. This is about the largest plate which can be mounted in the HC-6/U holder. Some of the mounting supports commonly used will not accept a plate as large as this. It is sometimes possible to obtain the advantage of a large diameter plate and yet restrict the distance between mounting points.

A method which has proved successful, at least in the few cases in which it was tried in this study, is to employ the so-called "racetrack" configuration. For example, a plate of .650 inch diameter has a segment ground off each end of the Z' diameter so that the lateral dimension of the plate in Z' direction is reduced to, say .600 inch. The plate can then be mounted along the narrow dimension. This dimension between the flat surfaces appears to be as critical as the plate diameter, so care must be taken to control it accurately, once a suitable dimension has been found.

TABLE 1

EXTRAPOLATED DESIGN PARAMETERS, PLANO-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters	Frequency KC	Plate Diameter	Curvature Diopters
1400	.600	14.0	1520	.552	15.2
	.585	14.0		.539	15.2
	.580	14.0		.534	15.2
1420	.592	14.2	1540	.545	15.4
	.577	14.2		.532	15.4
	.572	14.2		.527	15.4
1440	.583	14.4	1560	.538	15.6
	.569	14.4		.525	15.6
	.564	14.4		.520	15.6
1460	.575	14.6	1580	.531	15.8
	.561	14.6		.518	15.8
	.556	14.6		.514	15.8
1480	.567	14.8	1600	.525	16.0
	.553	14.8		.512	16.0
	.549	14.8		.508	16.0
1500	.560	15.0	1620	.518	16.2
	.546	15.0		.506	16.2
	.541	15.0		.501	16.2

TABLE 1 cont'd.

EXTRAPOLATED DESIGN PARAMETERS, PLANO-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters
1640	.603	17.3
	.512	16.4
	.499	16.4
1660	.602	10.0
	.596	17.5
	.506	16.6
1680	.595	10.1
	.589	17.7
	.500	16.8
1700	.588	10.2
	.582	17.9
	.494	17.0
1720	.581	10.3
	.575	18.2
	.488	17.2
1740	.600	9.2
	.574	10.4
	.568	18.4
	.483	17.4
1760	.593	9.3
	.568	10.6
	.562	18.6
	.477	17.6
1780	.587	9.4
	.561	10.7
	.556	18.8
	.472	17.8
1800	.580	9.5
	.555	10.8
	.550	19.0
	.467	18.0
1820	.574	9.6
	.549	10.9
	.543	19.2
	.462	18.2
1840	.567	9.7
	.543	11.0
	.538	19.4
	.456	18.4

TABLE 1 cont'd.

EXTRAPOLATED DESIGN PARAMETERS, PLANO-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters
1860	.561	9.8
	.537	11.2
	.532	19.6
	.452	18.6
1880	.555	9.9
	.531	11.3
	.526	19.8
1900	.550	10.0
	.525	11.4
	.521	20.0
1920	.544	10.1
	.520	11.5
	.515	20.3
1940	.538	10.2
	.515	11.6
	.510	20.5
1960	.533	10.3
	.510	11.8
	.505	20.7

Frequency KC	Plate Diameter	Curvature Diopters
1980	.527	10.4
	.505	11.9
	.500	20.9
2000	.600	12.0
	.500	12.0
	.495	21.2

TABLE 2

EXTRAPOLATED DESIGN PARAMETERS, BI-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters
800	.645	16.7
	.638	15.1
	.638	13.2
810	.645	14.5
	.637	17.0
	.630	15.2
820	.637	14.7
	.629	17.2
	.622	15.4
830	.640	11.2
	.630	14.8
	.614	15.6
840	.643	13.1
	.638	13.8
	.622	15.0
	.607	15.8
850	.641	11.9
	.635	13.2
	.615	15.2
	.600	16.0

Frequency KC	Plate Diameter	Curvature Diopters
860	.640	12.5
	.634	12.0
	.627	14.0
	.608	15.4
870	.632	12.7
	.626	12.2
	.616	14.3
	.600	15.6
880	.625	12.8
	.614	13.7
	.594	15.8
	.580	16.6
890	.618	12.9
	.612	12.5
	.606	14.5
	.587	15.9
900	.642	13.7
	.611	13.1
	.606	12.6
	.600	14.0
	.595	14.8

TABLE 2 cont'd.

EXTRAPOLATED DESIGN PARAMETERS, BI-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters	Frequency KC	Plate Diameter	Curvature Diopters
910	.635	13.9	950	.608	14.5
	.607	15.8		.581	16.5
	.604	13.6		.579	13.8
	.599	12.7		.574	13.3
	.560	17.1		.550	17.0
920	.628	14.0	960	.601	14.6
	.600	16.0		.575	16.7
	.598	13.4		.573	14.0
	.592	12.9		.568	13.4
	.586	14.3		.558	15.8
930	.621	14.2	970	.595	14.8
	.591	13.5		.567	14.1
	.586	13.0		.562	13.6
	.580	14.5		.556	15.1
940	.614	14.3	980	.619	12.3
	.585	13.7		.589	14.9
	.580	13.2		.561	14.2
	.573	15.4		.550	16.0

TABLE 2 cont'd.

EXTRAPOLATED DESIGN PARAMETERS, BI-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters
990	.583	15.1
	.556	14.4
	.551	13.9
	.545	15.4
1000	.578	15.2
	.550	18.0
	.550	15.0
	.550	14.6
	.545	14.0
	.540	15.6
1020	.566	15.5
	.539	14.8
	.534	14.3
1040	.555	15.8
	.529	15.1
	.524	14.6
1060	.545	16.2
	.519	15.4
	.514	14.8

Frequency KC	Plate Diameter	Curvature Diopters
1080	.611	9.9
	.535	16.5
	.509	15.7
	.505	15.1
1100	.600	10.1
	.525	16.8
	.500	16.0
	.495	15.4
1120	.589	10.3
	.527	17.9
	.516	17.1
1140	.639	6.9
	.579	10.4
	.507	17.4
1160	.628	7.0
	.569	10.6
	.498	17.7
1180	.618	7.1
	.559	10.8

TABLE 2 cont'd.

EXTRAPOLATED DESIGN PARAMETERS, BI-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters	Frequency KC	Plate Diameter	Curvature Diopters
1200	.642	11.1	1320	.625	10.6
	.607	7.2		.583	12.3
	.550	11.0		.552	8.0
				.500	12.1
1220	.631	11.3			
	.597	7.4	1340	.616	10.7
	.541	11.2		.575	12.4
				.543	8.1
1240	.621	11.5			
	.587	7.5	1360	.607	10.9
	.532	11.4		.566	12.6
				.535	8.2
1260	.611	11.7			
	.578	7.6	1380	.598	11.0
	.524	11.6		.558	12.8
				.528	8.3
1280	.602	11.9			
	.569	7.7	1400	.589	11.2
	.516	11.7		.550	13.0
				.520	8.4
1300	.635	10.4			
	.592	12.1	1420	.581	11.4
	.561	7.8		.542	13.2
	.508	11.9		.513	8.6

TABLE 2 cont'd.

EXTRAPOLATED DESIGN PARAMETERS, BI-CONVEX

Frequency KC	Plate Diameter	Curvature Diopters	Frequency KC	Plate Diameter	Curvature Diopters
1440	.573	11.5			
	.535	13.4			
	.506	8.7			
1460	.565	11.7			
	.527	13.6			
	.499	8.8			
1480	.557	11.8			
	.520	13.7			
1500	.550	12.0			
	.513	13.9			

V. CONCLUSIONS

The principal objective of this study was to obtain practical design data for fundamental mode AT type quartz resonators throughout the frequency range 800 kilocycles to 20 megacycles. The major portion of the effort involved obtaining suitable quartz plate configurations to meet the military specifications on equivalent motional resistance. Several different configurations were investigated, the most successful being plane-parallel, plano-convex, and bi-convex.

Plane-parallel plates proved satisfactory above 7 MC. This design is simple to fabricate and at the higher frequencies can have a lower equivalent resistance than fully contoured plates. It has a disadvantage from the point of view of the user in that the mode spectrum is quite complex near the fundamental frequency, sometimes causing slight shifts in resonant frequency with changes in oscillator tuning, drive level, or temperature.

Plano-convex plates have less sensitivity to mounting constraints and consequently can function well with smaller diameter to thickness ratios than plane-parallel plates. For d/t ratios greater than 27, a virtually constant equivalent resistance can be obtained throughout the operating temperature range. Plano-convex plates also have an advantage if high values of Q are desired, since the equivalent motional inductance tends to be higher than that obtained with uncontoured plates. The frequency shifts characteristic of plane-parallel plates are not experienced with fully contoured plates unless high curvatures or large electrodes are employed. Where resonant frequency shifts do occur, the magnitude of the shift is much greater than that of uncontoured plates. Excellent results were obtained with plano-convex plates from 3 MC to 8 MC and the range could probably be extended to higher frequencies if desired. Satisfactory results were achieved with this design down to a frequency of 1.4 MC, but the decreasing diameter to thickness ratio tends to degrade performance somewhat.

Bi-convex plates were produced which would meet the performance specifications down to a frequency of 850 KC and designs were calculated down to 800 KC, employing slightly larger plate diameters than those used in this study. The bi-convex configuration performs similarly to the plano-convex,but appears to be somewhat less affected by parasitic modes at low values of diameter to thickness ratio. In some cases, the increase in equivalent resistance when going from series to parallel resonant operation is greater than that expected, but the reason for this is not definitely known.

The choice of a quartz plate configuarion will depend on a number of other factors as well as the resonant frequency and maximum possible plate dimensions. Plane-parallel plates offer the greatest economy of fabrication, but for high stability requirements, the plano-convex design is probably a better choice, even at the upper portion of the fundamental mode range. The inherent difficulties associated with thickness-shear plates with diameter to thickness ratios of less than about 17 make their use somewhat questionable. Although it is possible to produce resonators of these dimensions which meet specifications, their fabrication requires extreme care, if consistent results are to be achieved. For frequencies below 2 MC, the HC-6/U holder is just too small to permit good resonator characteristics. Where plates of small d/t ratio must be used, series resonant operation, coupled with very low drive levels, is to be recommended.

VI. TABLES OF EXPERIMENTAL DATA

The following tables list the temperature-run measurements that were made on the experimental resonators fabricated for this study. Each entry gives the results of a group of resonators with a common set of physical dimensions and with approximately the same resonant frequency. The plate diameter and the electrode diameter are in inches. The mounting axis column gives the direction of the line between the two mounting points. The symbol "R" refers to a random mounting orientation. All of the plano-convex and bi-convex units shown were mounted on the Z' axis. The curvature of these units is given in diopters. A table converting radius of curvature to diopters appears in Appendix C. The shunt capacitance, C_o, is the measured value in picofarads. The last six columns list the equivalent series resistance measurements within the specified temperature range. Minimum and maximum refer to the limits measured within the temperature range. For example, in the upper curve of Figure 2, the minimum and maximum values would be approximately 5 ohms and 35 ohms respectively. The "low" and "high" columns give the lowest and highest values of these resistance limits attained by any unit in the group. The "mean" columns list the mean values of the minimum and maximum resistances for the group. The letters "N.O." indicate that there was no oscillation.

TABLE 3

UNCONTOURED

Freq KC	Plate Dia	Elec Dia	Mt Axis	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
5018	.550	.350	Z'	5	8.7	39	52	64	110	149	190
5005	.550	.310	Z'	5	6.9	27	36	47	58	80	135
5014	.550	.250	Z'	5	4.7	78	86	95	150	182	245
5011	.550	.210	Z'	5	3.7	59	74	108	135	204	420
5143	.550	.280	R	5	5.7	40	56	88	68	88	130
5150	.520	.280	R	4	5.7	64	68	80	120	168	280
5613	.550	.310	R	5	7.5	50	56	70	120	146	185
5613	.520	.310	R	4	7.5	82	87	92	110	123	135
6112	.550	.260	R	4	6.0	7.0	8.7	11	30	39	45
6110	.520	.260	R	5	6.0	16	30	42	40	55	97
6309	.550	.280	X	5	7.0	7.2	8.6	9.2	20	32	46
6288	.490	.280	X	5	7.0	16	21	26	37	39	40
6302	.450	.280	X	5	7.0	22	23	27	51	70	98
6954	.550	.250	X	11	6.1	6.5	9.6	17	12	20	30
6961	.520	.260	R	4	6.7	6.5	7.8	9.5	15	23	36
6957	.515	.260	R	5	6.7	7.5	10	16	18	21	24
6956	.490	.260	R	2	6.7	15	18	24	32	34	36
6957	.480	.260	R	3	6.7	14	19	27	28	33	43
6963	.460	.260	R	4	6.7	36	41	46	63	74	85
6968	.430	.260	R	5	6.7	55	74	95	100	130	160

TABLE 3 cont'd.

UNCONTOURED

Freq KC	Plate Dia	Elec Dia	Mt Axis	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
7151	.550	.250	X	11	6.4	15	21	31	31	39	51
7133	.550	.210	X	5	4.8	25	34	43	32	73	120
7119	.490	.250	X	5	6.4	8.0	14	24	20	24	32
7142	.490	.250	X	11	6.4	7.0	12	22	16	21	50
7134	.450	.250	X	5	6.4	24	30	43	28	45	63
7311	.375	.250	Z'	5	6.5	55	77	99	133	144	160
7544	.550	.250	X	11	6.6	12	16	19	14	22	31
7531	.550	.210	X	10	4.9	16	19	30	22	33	56
7542	.490	.250	X	11	6.6	11	20	38	17	28	48
7545	.450	.250	X	10	6.6	27	41	68	53	74	94
7996	.550	.400	Z'	3	17.0	2.1	2.5	3.8	3.5	4.1	5.0
8008	.550	.350	Z'	3	13.2	2.7	3.0	3.6	3.7	5.1	10
8031	.550	.310	X	10	10.6	2.2	2.8	3.7	3.0	3.5	4.5
8008	.550	.250	Z'	2	7.1	4.1	4.5	5.1	13	14	15
8007	.550	.250	X	3	7.1	4.5	5.8	7.3	12	12	12
8051	.550	.210	Z'	5	5.5	12	14	24	16	20	27
8052	.550	.210	X	5	5.5	17	20	24	21	24	28
8016	.550	.180	Z'	3	4.3	6.2	6.6	7.3	30	32	34
8017	.550	.180	X	2	4.3	11	13	15	18	19	22
8000	.490	.350	Z'	2	13.2	3.3	3.9	4.7	7.5	7.6	7.8
7995	.490	.350	X	3	13.2	5.0	6.2	12	7.3	12	23
8011	.490	.250	Z'	2	7.1	4.3	5.1	6.2	15	17	21

TABLE 3 cont'd.

UNCONTOURED

Freq KC	Plate Dia	Elec Dia	Mt Axis	No. of Units	C_o	Series Resistance (-55° to +90°C) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
8001	.490	.250	X	4	7.1	5.7	6.3	6.8	9.8	14	20
8046	.490	.210	Z'	5	5.2	9.0	11	12	19	19	23
8041	.490	.210	X	5	5.2	19	33	31	36	41	50
8020	.490	.180	Z'	3	4.2	7.2	7.6	8.0	22	24	27
8010	.490	.180	X	3	4.2	11	12	13	19	19	20
8006	.450	.250	Z'	2	7.1	5.4	5.7	6.0	21	21	22
8008	.450	.250	X	2	7.1	6.4	6.6	6.8	17	18	18
8045	.450	.210	Z'	5	5.1	11	15	19	37	39	40
8050	.450	.210	X	11	5.1	30	50	74	38	60	84
8000	.450	.180	Z'	2	4.1	7.0	7.3	7.7	26	30	34
8010	.450	.180	X	3	4.1	8.9	13	20	27	37	50
8001	.400	.250	Z'	5	7.2	10	16	24	26	32	46
8023	.400	.250	X	5	7.2	12	13	16	60	81	100
7987	.375	.250	Z'	5	7.2	20	36	67	77	97	110
8011	.375	.250	X	4	7.2	16	19	24	56	70	80
9049	.550	.230	X	5	6.8	4.2	5.0	5.4	5.0	7.0	23
9061	.550	.198	X	10	5.3	5.0	6.0	22	8.5	12	24
9063	.550	.186	X	10	4.9	6.5	7.0	9.4	9.5	15	25
9059	.550	.160	X	10	3.8	7.5	10	12	16	22	38
9050	.490	.230	X	5	6.8	6.0	7.1	8.8	8.0	10	12
9054	.490	.198	X	9	5.2	7.8	8.6	11	12	17	22
9052	.490	.186	X	10	4.8	8.0	9.2	13	10	18	31
9055	.490	.160	X	10	3.8	11	13	14	20	29	39

TABLE 3 cont'd.

UNCONTOURED

Freq KC	Plate Dia	Elec Dia	Mt Axis	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
8988	.400	.230	Z'	5	6.8	6.0	6.8	7.5	14	16	17
8986	.375	.230	Z'	5	6.8	6.8	7.6	8.1	16	17	18
10,052	.550	.250	Z'	3	8.5	2.3	3.0	7.0	38	39	41
9,976	.550	.210	X	5	6.3	4.8	6.0	11	12	22	52
10,045	.550	.198	Z'	4	5.6	4.7	5.0	5.1	14	19	29
10,072	.550	.198	X	8	5.6	5.0	6.0	7.0	36	43	50
10,049	.550	.160	Z'	5	4.0	7.0	10	28	18	28	110
10,011	.490	.210	X	3	6.3	5.6	6.0	6.8	8.2	9.0	9.0
10,080	.490	.198	X	11	5.6	7.0	9.0	12	10	12	16
9,995	.450	.210	X	3	6.3	6.0	6.0	7.5	9.5	11	13
10,076	.450	.198	X	10	5.6	9.8	11	12	12	14	18
10,024	.400	.210	X	3	6.3	6.0	6.0	8.0	7.7	8.0	10
10,094	.400	.198	X	10	5.6	19	25	31	26	31	38
11,011	.490	.210	X	5	7.0	4.2	4.4	4.7	6.3	7.0	10
11,057	.490	.198	X	5	6.1	3.2	3.4	3.5	3.9	6.1	14
11,004	.450	.210	X	4	7.0	3.9	5.2	9.3	5.8	7.2	8.5
11,054	.450	.198	X	5	6.1	4.2	4.5	5.3	5.4	6.3	7.5
10,976	.400	.210	Z'	5	6.9	5.2	6.3	9.6	9.0	11	18
11,013	.400	.210	X	3	7.0	4.3	4.6	5.3	6.0	7.6	10
11,062	.400	.198	X	6	6.1	7.0	8.7	12	8.8	11	16
11,013	.375	.210	Z'	3	6.9	5.1	5.8	6.5	9.4	11	13

TABLE 3 cont'd.

UNCONTOURED

Freq KC	Plate Dia	Elec Dia	Mt Axis	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
11,962	.550	.310	Z'	9	15.2	3.0	3.8	5.2	3.5	4.5	6.5
11,980	.550	.250	Z'	11	10.3	3.5	3.9	5.4	3.8	5.0	11
11,970	.550	.210	Z'	10	7.6	4.0	4.9	7.0	5.0	7.0	12
11,989	.550	.160	Z'	5	4.8	2.5	4.4	7.8	13	16	25
12,020	.550	.100	Z'	9	2.3	13	15	17	38	81	150
11,953	.500	.310	Z'	4	15.1	2.9	3.2	3.5	4.5	5.1	5.7
11,982	.500	.250	Z'	3	10.2	3.5	4.0	4.3	4.1	4.5	4.9
11,955	.500	.210	Z'	2	7.5	4.4	4.4	4.5	5.6	5.9	6.2
11,993	.500	.160	Z'	3	4.6	6.5	6.7	6.9	14	15	16
12,019	.500	.100	Z'	4	2.2	14	15	17	29	37	70
11,957	.450	.310	Z'	4	15.0	2.8	3.0	32	3.6	3.7	3.8
11,987	.450	.250	Z'	3	10.2	3.8	4.1	4.5	4.5	5.0	5.9
11,984	.450	.160	Z'	4	4.6	6.7	7.0	7.3	13	14	15
12,009	.450	.100	Z'	3	2.2	15	15	16	24	32	92
12,593	.550	.210	X	10	7.8	5.0	5.8	10	6.0	7.9	16
12,579	.550	.186	X	10	6.3	6.0	6.3	7.6	9.9	13	17
12,579	.550	.160	X	10	4.9	8.5	9.7	11	12	16	24
14,982	.550	.250	Z'	10	12.6	2.7	2.9	3.3	3.6	7.2	12
15,054	.550	.180	Z'	5	7.3	3.5	3.7	4.3	5.2	6.0	7.9
15,011	.550	.150	Z'	9	5.2	4.7	5.0	5.3	6.8	15	23
15,072	.550	.100	Z'	5	2.6	9.4	12	15	17	21	26
16,685	.375	.160	R	10	6.3	4.5	5.4	7.0	5.5	7.7	14

TABLE 3 cont'd.

UNCONTOURED

Freq KC	Plate Dia	Elec Dia	Mt Axis	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
17,974	.375	.150	R	9	6.0	6.3	7.1	8.0	8.8	9.8	12
19,966	.375	.150	R	16	6.3	7.5	8.4	11.5	9.3	11	15
22,202	.375	.250	Z'	3	18.2	4.8	5.2	6.0	5.5	6.3	9.0
22,351	.375	.160	Z'	6	7.8	3.0	4.4	8.6	3.7	5.2	9.5
22,350	.375	.100	Z'	4	3.5	5.5	8.0	14	8.7	12	17

TABLE 4

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No. of Units	C_o	Series Resistance (-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1402	.600	.400	14	5	4.2	40	47	55	74	100	150
1403	.600	.400	12	5	4.1	55	71	83	110	130	200
1405	.600	.400	10	5	4.1	40	51	76	120	210	220
1402	.600	.400	8	4	4.0	200	280	590	3.8k	-	N.O.
1409	.600	.400	6	2	4.0	210	350	1.1k	1.2k	-	N.O.
1403	.595	.400	14	3	4.2	61	72	90	170	210	240
1405	.595	.400	12	3	4.1	58	70	90	210	380	1.9k
1404	.595	.400	10	2	4.1	85	130	310	1.8k	-	N.O.
1401	.595	.400	8	3	4.0	170	200	300	500	720	1.6k
1403	.590	.400	14	2	4.2	59	64	70	770	850	940
1401	.590	.400	12	3	4.1	120	150	260	610	790	1.8k
1404	.590	.400	10	3	4.1	90	130	160	210	350	760
1402	.590	.400	8	3	4.0	310	450	1.2k	780	1.2k	3.5k
1403	.585	.400	14	2	4.2	65	75	88	140	140	140
1401	.585	.400	12	3	4.1	130	140	140	220	240	300
1404	.585	.400	10	3	4.1	240	280	300	600	-	N.O.
1400	.585	.400	8	2	4.0		N.O.			N.O.	
1403	.580	.400	14	3	4.2	76	88	110	160	180	240
1405	.580	.400	12	3	4.1	420	440	470	1.1k	-	N.O.
1404	.580	.400	10	3	4.1	280	330	400	880	940	960
1400	.580	.400	8	1	4.0		275			N.O.	
1401	.550	.400	26	4	4.4	110	150	230	280	340	450
1400	.550	.400	24	5	4.4	180	360	1.0k	4.4k	-	N.O.
1405	.550	.400	22	4	4.3	200	240	310	570	950	1.1k

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1397	.550	.400	20	6	4.2	160	240	450	360	650	1.1k
1398	.550	.400	18	5	4.2	120	140	200	520	650	900
1400	.550	.400	16	5	4.1	90	110	130	460	820	1.6k
1398	.550	.400	14	5	4.1	120	150	250	210	330	430
1399	.550	.400	12	5	4.0	90	170	310	150	340	900
1400	.550	.400	10	3	4.0	68	95	190	160	230	750
1551	.550	.400	20	6	4.7	160	260	480	2.0k	-	N.O.
1601	.550	.400	22	3	5.0	210	230	260	500	-	N.O.
1621	.550	.400	20	4	4.9	90	130	200	200	260	670
1604	.550	.400	18	4	4.8	110	120	170	190	280	360
1611	.550	.400	16	5	4.7	85	110	190	180	270	760
1605	.550	.400	14	3	4.5	75	80	90	150	190	480
1600	.550	.400	12	5	4.6	140	140	150	250	-	N.O.
1602	.550	.400	10	4	4.3	90	110	190	140	-	N.O.
1602	.550	.400	8	4	4.3	85	100	130	180	210	260
1604	.550	.400	6	5	4.2	160	200	240	400	470	580
1805	.550	.400	20	3	5.6	35	41	50	180	180	200
1800	.550	.400	19	3	5.4	42	47	50	79	90	99
1799	.550	.400	18	6	5.4	46	57	75	95	110	140
1801	.550	.400	17	2	5.4	45	46	47	98	100	110
1802	.550	.400	16	2	5.3	42	55	80	150	180	220
1798	.550	.400	14	4	5.2	38	43	46	120	190	620

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance (-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1803	.550	.400	12	5	5.1	50	69	90	120	220	500
1801	.550	.400	10	5	4.9	60	74	120	190	330	470
1799	.550	.400	8	6	4.9	63	79	110	140	170	240
1904	.550	.400	18	4	5.7	70	93	130	160	200	330
1901	.550	.400	16	5	5.5	66	89	120	250	410	1.0k
1901	.550	.400	14	5	5.4	53	57	60	140	180	270
1905	.550	.400	12	5	5.2	50	55	65	120	140	160
1904	.550	.400	10	3	5.2	60	67	76	110	140	210
1905	.550	.400	8	5	5.1	65	70	81	110	130	160
1904	.550	.400	6	4	5.0	130	160	220	600	820	1.2k
1903	.550	.400	4	5	4.9	65	120	290	160	470	1.5k
2002	.600	.350	12	3	4.4	30	33	36	89	93	98
2002	.600	.350	10	4	4.4	42	50	61	110	160	200
2001	.600	.350	8	5	4.3	64	69	80	160	200	230
2000	.600	.350	6	5	4.2	57	72	83	130	150	170
2003	.600	.350	4	5	4.2	33	39	51	70	110	270
2004	.590	.350	12	6	4.4	38	44	52	90	100	130
2000	.590	.350	8	5	4.3	60	68	77	120	120	130
2003	.590	.350	4	7	4.2	45	70	110	600	730	1.1k
2004	.580	.350	12	6	4.4	35	40	50	120	130	140
2001	.580	.350	8	5	4.3	45	52	65	150	160	180
2002	.580	.350	4	6	4.2	110	140	250	330	410	620

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to $+90^{\circ}C$) Minimum Low	Mean	High	Maximum Low	Mean	High
2004	.575	.350	12	6	4.4	43	45	50	100	110	120
2001	.575	.350	8	4	4.3	59	66	79	140	150	160
2002	.575	.350	4	4	4.2	60	74	130	80	120	240
2004	.570	.350	12	6	4.4	46	52	55	96	120	140
2001	.570	.350	8	4	4.3	53	57	60	130	140	150
2002	.570	.350	4	3	4.2	95	160	270	340	450	650
2004	.565	.350	12	6	4.4	51	61	69	86	260	1.3k
2001	.565	.350	8	4	4.3	55	59	64	150	150	150
2003	.565	.350	4	3	4.2	55	79	110	120	150	280
1998	.550	.350	16	4	4.4	39	42	48	160	170	180
2010	.550	.350	14	5	4.3	30	41	59	140	150	190
2012	.550	.350	13	5	4.2	52	62	78	150	180	230
1993	.550	.350	12	5	4.2	75	83	90	120	140	160
2000	.550	.350	11	5	4.2	45	54	66	120	140	180
1995	.550	.350	10	5	4.2	43	48	52	180	190	220
2003	.550	.350	9	5	4.1	64	70	78	100	130	160
1996	.550	.350	8	5	4.2	70	77	82	180	180	190
2005	.550	.350	7	4	4.1	43	51	65	180	190	200
2016	.550	.350	6	5	4.1	44	65	90	110	130	170
2000	.550	.350	5	5	4.1	110	130	180	230	310	370
1996	.550	.350	4	4	4.1	77	89	120	200	410	740
2003	.550	.350	3	1	4.1		1.1k			2.3k	
2005	.550	.350	2.5	2	3.9	230	380	1.2k	2.3k	-	N.O.
2000	.550	.350	2	1	4.0		1.2k			N.O.	

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No. of Units	C_o	Series Resistance (-55^o to $+90^oC$) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
2005	.545	.350	16	3	4.4	42	58	75	160	190	220
2000	.545	.350	12	5	4.3	51	70	85	140	170	210
2000	.545	.350	8	3	4.2	60	70	100	160	180	210
2000	.545	.350	4	3	4.1	27	36	68	44	69	140
2005	.540	.350	16	3	4.4	49	51	53	160	170	170
1994	.540	.350	12	5	4.3	58	80	110	85	120	200
2000	.540	.350	8	3	4.2	52	58	68	130	140	140
2000	.540	.350	4	3	4.1	320	330	360	730	840	1.0k
1999	.500	.350	20	5	4.6	50	71	90	140	200	300
1999	.500	.350	16	5	4.5	42	59	78	150	220	350
2020	.500	.350	14	5	4.5	58	72	84	150	160	170
2000	.500	.350	12	5	4.4	45	54	70	110	120	140
2002	.500	.350	12	5	4.2	63	70	77	100	110	120
1999	.500	.350	10	5	4.3	60	75	100	150	170	210
2004	.500	.350	8	5	4.1	110	180	300	250	540	1.1k
2100	.550	.350	14	3	4.6	59	61	63	160	170	180
2105	.550	.350	12	4	4.5	37	48	58	110	120	140
2105	.550	.350	10	4	4.5	63	68	77	180	190	200
2103	.550	.350	8	3	4.4	61	62	64	180	190	200
2104	.550	.350	6	4	4.4	62	74	85	120	120	130
2106	.550	.350	4	4	4.3	39	57	120	70	110	210

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
2253	.550	.350	14	5	4.9	57	64	70	170	190	200
2253	.550	.350	13	4	4.9	58	68	77	160	180	190
2254	.550	.350	12	5	4.8	38	44	55	130	160	200
2250	.550	.350	11	5	4.7	37	42	60	100	110	120
2252	.550	.350	10	6	4.6	40	46	54	200	220	240
2256	.550	.350	9	2	4.6	75	78	82	150	170	190
2253	.550	.350	8	7	4.5	48	60	80	130	160	180
2250	.550	.350	7	6	4.6	33	36	51	130	150	170
2249	.550	.350	6	2	4.4	27	29	32	52	60	71
2252	.550	.350	5	5	4.6	13	18	25	28	38	49
2251	.550	.350	4	5	4.6	30	47	65	60	140	300
2250	.550	.350	3	5	4.4	62	100	160	990	1.4k	2.0k
2501	.550	.350	12	5	5.4	52	59	70	190	210	230
2502	.550	.350	11	5	5.3	52	63	77	150	-	N.O.
2503	.550	.350	10	5	5.2	41	46	50	150	160	180
2502	.550	.350	9	5	5.2	44	48	53	160	170	180
2501	.550	.350	8	4	5.1	49	53	57	120	140	140
2501	.550	.350	7	5	5.1	34	39	50	65	70	82
2503	.550	.350	6	5	5.0	21	23	25	33	41	60
2505	.550	.350	5.5	5	5.0	19	20	22	23	25	26
2503	.550	.350	5.0	7	4.9	18	19	22	20	22	25
2504	.550	.350	4.5	8	4.9	17	18	20	21	23	28
2503	.550	.350	3.5	5	4.9	21	24	27	38	42	50
2503	.550	.350	3.0	5	4.9	25	29	37	94	140	250

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
2503	.550	.350	2.5	5	4.8	28	40	65	150	210	430
2501	.550	.350	2.0	5	4.8	36	59	100	150	180	270
2503	.500	.350	12	3	5.4	63	73	90	140	150	180
2526	.500	.350	10	5	5.3	51	63	75	110	140	160
2503	.500	.350	8.0	3	5.2	44	46	47	83	84	86
2501	.500	.350	6.0	5	5.1	34	38	48	49	57	72
2500	.500	.350	5.0	5	5.0	22	26	38	45	53	68
2497	.500	.350	4.0	5	5.0	19	23	29	28	33	44
2500	.500	.350	3.0	5	4.9	34	49	72	68	79	94
2757	.550	.350	6.0	4	5.3	22	25	29	34	35	38
2754	.550	.350	5.5	5	5.4	17	19	21	23	26	28
2758	.550	.350	5.0	4	5.4	18	18	19	21	22	24
2754	.550	.350	4.5	6	5.3	16	17	17	21	22	24
2754	.550	.350	4.0	5	5.3	14	16	21	20	22	26
2753	.550	.350	3.5	5	5.3	16	18	20	20	24	30
2758	.550	.350	3.0	6	5.3	13	15	22	20	24	27
3003	.600	.350	6.0	4	5.8	16	17	19	24	27	40
3000	.600	.350	4.5	5	5.7	12	15	18	19	22	26
3001	.600	.350	3.0	5	5.6	8.0	11	14	13	16	21
2997	.600	.350	2.25	6	5.6	9.0	13	18	92	98	110
2998	.600	.350	1.5	6	5.6	8.5	9.1	10	12	14	16
3002	.550	.350	9.0	6	6.1	23	30	35	81	100	120
3004	.550	.350	7.5	6	5.8	28	32	47	48	74	140

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
3006	.550	.350	6.0	5	5.8	16	19	21	24	25	26
3002	.550	.350	5.0	5	5.7	13	15	16	18	20	25
3004	.550	.350	4.5	2	5.7	14	15	16	18	20	22
3001	.550	.350	4.0	3	5.6	13	14	16	15	17	18
3002	.550	.350	3.5	6	5.6	12	13	15	16	17	19
3005	.550	.350	3.0	5	5.7	12	12	14	18	20	21
3002	.550	.350	2.5	5	5.7	13	16	18	75	98	120
3001	.500	.350	10	5	6.2	32	40	50	140	170	210
3002	.500	.350	8.0	5	6.1	29	30	35	67	69	76
3000	.500	.350	6.0	5	5.9	15	16	18	25	26	28
2999	.500	.350	5.0	5	5.8	13	14	16	20	22	27
3003	.500	.350	4.0	5	5.7	14	15	17	20	21	21
3000	.500	.350	3.0	5	5.6	13	16	19	28	31	38
3003	.450	.250	12	5	3.4	63	75	90	160	230	380
3002	.450	.250	9.0	6	3.3	40	44	48	140	150	160
3003	.450	.250	6.0	5	3.3	21	26	30	40	44	48
3001	.450	.250	3.0	5	3.2	54	99	300	280	480	620
3998	.550	.310	6.25	5	6.3	18	22	24	30	36	42
3997	.550	.310	5.0	5	6.2	14	18	23	25	32	42
4003	.550	.310	4.5	4	6.2	16	17	18	34	38	41
3997	.550	.310	4.0	5	6.1	16	17	19	24	31	54
4006	.550	.310	3.5	5	6.3	14	16	18	40	43	49
4000	.550	.310	3.0	5	6.2	21	23	27	60	75	150

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
3981	.550	.310	2.5	5	6.1	11	14	20	23	30	38
4000	.550	.310	2.25	4	6.0	10	12	14	11	15	20
4045	.550	.310	2.0	5	6.0	10	11	11	11	11	12
4003	.550	.310	2.0	5	5.8	6.6	7.2	8.2	8.5	8.9	9.5
4005	.550	.310	1.75	5	6.1	8.5	9.8	11	10	12	16
4004	.550	.310	1.5	5	6.0	8.5	9.8	11	10	12	18
4006	.550	.310	1.37	4	5.8	12	13	15	20	22	26
4004	.550	.310	1.0	4	5.8	17	18	20	22	25	30
3994	.500	.310	4.0	5	5.9	15	16	18	21	23	25
4005	.500	.310	2.0	5	5.8	7.5	8.1	9.9	7.9	9.3	12
4001	.490	.310	2.0	5	5.9	11	12	15	12	13	18
3996	.450	.310	4.0	5	5.9	14	15	16	25	27	32
4003	.450	.310	2.0	5	5.7	9.0	10	12	10	12	16
5003	.500	.250	2.5	5	4.9	8.0	9.6	11	9.0	12	20
5000	.450	.250	2.5	5	4.8	6.0	8.8	12	11	12	14
5005	.400	.250	2.5	5	4.7	9.8	10	11	11	16	32
6006	.550	.280	3.0	5	7.1	8.0	8.7	9.0	14	15	18
6008	.550	.250	3.0	5	5.7	8.0	8.7	11	12	14	28
6008	.550	.210	3.0	5	4.3	9.0	9.2	10	11	12	13
6012	.550	.180	3.0	5	3.4	10	10	10	12	15	21
6008	.400	.280	3.0	5	7.1	7.0	8.7	10	10	13	16
6006	.400	.250	3.0	5	5.6	8.0	9.0	10	11	15	21
6007	.400	.210	3.0	5	4.3	8.0	8.9	10	12	16	29
6008	.400	.180	3.0	5	3.5	9.9	11	13	12	14	20

TABLE 4 cont'd.

PLANO-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
7017	.550	.180	3.5	5	3.9	11	14	18	14	16	20
6968	.550	.160	3.5	5	3.2	14	14	15	17	19	26
7013	.400	.250	3.5	5	6.6	9.4	10	12	13	18	28
7012	.400	.210	3.5	5	4.8	9.5	10	12	10	13	15
7014	.400	.180	3.5	5	3.9	9.7	11	15	16	18	30
7014	.400	.160	3.5	5	3.2	10	12	14	16	24	38
8028	.400	.210	4.0	5	5.5	9.5	10	12	14	15	19
8032	.400	.180	4.0	5	4.4	10	10	11	12	13	14
8033	.400	.160	4.0	5	3.7	9.3	10	11	12	15	44
8036	.400	.140	4.0	5	3.1	10	11	12	12	16	41

TABLE 5

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
800	.600	.400	20	3	2.8	700	740	800		N.O.	
800	.600	.400	18	3	2.8	430	580	1.0k	1.3k	-	N.O.
800	.600	.400	16	6	2.8	380	620	1.2k	1.5k	-	N.O.
801	.600	.400	14	6	2.7		N.O.			N.O.	
803	.600	.400	12	8	2.7		N.O.			N.O.	
801	.550	.400	24	2	2.9	580	810	1.4k		N.O.	
801	.550	.400	22	6	2.9		N.O.			N.O.	
807	.550	.400	21	5	2.9		N.O.			N.O.	
801	.550	.400	20	6	2.8		N.O.			N.O.	
804	.550	.400	19	7	2.8		N.O.			N.O.	
820	.600	.400	18	3	3.1	210	270	330	800	-	N.O.
822	.600	.400	16	6	3.1		N.O.			N.O.	
821	.600	.400	14	4	3.0		N.O.			N.O.	
821	.600	.400	12	5	3.0	450	-	N.O.	2.0k	-	N.O.
822	.550	.400	22	6	2.9		N.O.			N.O.	
822	.550	.400	21	5	2.9		N.O.			N.O.	
820	.550	.400	20	6	2.9		N.O.			N.O.	
822	.550	.400	19	6	2.8		N.O.			N.O.	
824	.550	.400	16	6	2.8		N.O.			N.O.	
842	.600	.400	18	3	3.1	140	150	180	510	-	N.O.
841	.600	.400	16	4	3.1	130	150	180	360	590	880
842	.600	.400	14	3	3.0	130	180	340	340	-	N.O.

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
841	.600	.400	12	3	2.9	310	440	980	1.2k	-	N.O.
840	.550	.400	22	6	3.0		N.O.			N.O.	
845	.550	.400	18	6	3.0		N.O.			N.O.	
850	.600	.400	16	3	3.0	85	100	150	130	200	720
850	.600	.400	14	6	3.0	90	120	190	170	250	530
850	.600	.400	12	4	2.9	180	200	270	350	600	1.0k
851	.550	.400	22	6	3.0	1.4k	-	N.O.		N.O.	
849	.550	.400	20	8	3.0	700	-	N.O.		N.O.	
851	.550	.400	18	3	3.0	310	440	680		N.O.	
849	.550	.400	16	5	2.9	190	350	900		N.O.	
856	.600	.400	16	4	3.0	85	98	110	200	230	290
860	.600	.400	18	6	3.1	92	100	110	170	210	340
862	.600	.400	16	5	3.0	100	120	160	400	610	880
862	.600	.400	14	5	3.0	92	140	220	320	570	1.0k
862	.600	.400	12	5	2.9	140	190	350	460	-	N.O.
859	.550	.400	20	3	3.0	200	360	590	570	950	1.4k
865	.550	.400	22	6	2.9		N.O.			N.O.	
865	.550	.400	20	7	2.9		N.O.			N.O.	
866	.550	.400	18	6	2.9		N.O.			N.O.	
865	.550	.400	16	6	3.0		N.O.			N.O.	

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55^o to $+90^oC$) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
869	.550	.400	20	3	3.1	280	350	550	1.1k	1.3k	1.8k
875	.550	.400	21	3	3.1	480	700	1.5k		N.O.	
874	.550	.400	20	4	3.0	320	580	1.3k	1.4k	-	N.O.
876	.550	.400	19	4	3.0	210	260	290	2.0k	-	N.O.
874	.550	.400	18	6	2.9	1.2k	-	N.O.		N.O.	
876	.550	.400	17	2	2.9	380	500	750		N.O.	
876	.550	.400	16	2	3.0	530	780	1.4k		N.O.	
875	.500	.400	21	4	3.0		N.O.			N.O.	
874	.500	.400	20	6	3.0		N.O.			N.O.	
875	.500	.400	19	7	3.0		N.O.			N.O.	
874	.500	.400	18	6	3.0		N.O.			N.O.	
876	.500	.400	17	6	3.0		N.O.			N.O.	
875	.500	.400	16	5	2.9		N.O.			N.O.	
882	.600	.400	16	3	3.0	100	120	150	350	480	840
882	.600	.400	14	5	2.9	85	97	140	200	340	1.8k
882	.600	.400	12	3	2.9	79	90	120	210	250	360
879	.550	.400	20	3	3.1	220	360	650		N.O.	
890	.600	.400	18	3	3.0	140	150	180	440	500	650
890	.550	.400	20	2	3.1	240	280	340	920	1.0k	1.2k
901	.600	.400	18	6	3.2	120	140	170	220	300	490
900	.600	.400	16	6	3.2	76	89	110	180	-	N.O.
901	.600	.400	14	6	3.1	74	84	110	100	150	260

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55^o to $+90^oC$) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
901	.600	.400	12	4	3.1	160	170	250	360	590	1.3k
899	.550	.400	21	5	3.2	140	210	300	760	1.3k	2.0k
900	.550	.400	20	2	3.1	170	190	220		N.O.	
903	.550	.400	19	3	3.2	110	130	160	2.5k	-	N.O.
901	.550	.400	18	3	3.1	83	140	520	200	-	N.O.
902	.550	.400	18	4	3.0	120	270	720	460	-	N.O.
904	.550	.400	16	3	3.0	230	380	700		N.O.	
900	.550	.400	15	7	3.1		N.O.			N.O.	
900	.550	.400	14	5	3.1		N.O.			N.O.	
900	.550	.400	13	7	3.0		N.O.			N.O.	
921	.600	.400	16	5	3.1	83	96	110	130	170	280
922	.600	.400	14	5	3.0	72	98	140	180	230	320
921	.600	.400	12	4	3.0	85	98	110	180	230	480
924	.550	.400	20	7	3.2	95	120	140	360	440	530
924	.550	.400	19	6	3.2	94	120	160	190	270	320
925	.550	.400	18	5	3.0	85	110	170	310	390	540
926	.550	.400	17	5	3.2	100	120	150	180	240	280
926	.550	.400	16	7	3.2	110	120	150	250	330	500
941	.600	.400	16	5	3.2	85	100	120	170	-	N.O.
946	.600	.400	14	5	3.1	90	110	150	280	440	1.4k
943	.600	.400	12	3	3.0	140	270	580	1.8k	-	N.O.

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
952	.600	.400	16	4	3.2	81	91	100	160	230	500
950	.600	.400	14	4	3.2	95	130	270	210	-	N.O.
950	.600	.400	12	5	3.1	83	120	200	170	430	1.3k
950	.550	.400	19	4	3.3	90	93	95	190	270	380
949	.550	.400	18	5	3.3	87	110	160	200	450	800
949	.550	.400	17	6	3.2	72	84	170	140	210	300
950	.550	.400	16	5	3.2	76	95	170	93	250	960
950	.550	.400	15	5	3.1	100	130	170	240	360	920
951	.550	.400	14	5	3.0	130	170	290	270	-	N.O.
952	.550	.400	13	4	3.1	170	230	320	1.1k	-	N.O.
951	.550	.400	12	5	3.0	120	220	740	1.0k	-	N.O.
961	.600	.400	16	5	3.2	95	98	100	180	220	320
962	.600	.400	14	5	3.1	94	190	550	360	-	N.O.
962	.600	.400	12	5	3.1	260	300	380	1.4k	-	N.O.
960	.550	.400	19	4	3.3	97	120	190	260	380	600
961	.550	.400	18	4	3.3	90	120	160	180	280	520
961	.550	.400	17	5	3.3	85	94	110	230	350	520
961	.550	.400	16	3	3.3	77	89	110	760	-	N.O.
962	.550	.400	15	4	3.2	80	90	110	170	540	1.5k
961	.550	.400	14	4	3.2	100	110	120	230	400	1.2k
961	.550	.400	13	3	3.2	87	110	180	310	543	1.2k
962	.550	.400	12	4	3.1	78	94	110	240	380	900

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No. of Units	C_o	Minimum Low	Minimum Mean	Minimum High	Maximum Low	Maximum Mean	Maximum High
974	.550	.400	19	4	3.3	110	140	260	170	270	660
974	.550	.400	18	3	3.3	87	100	150	230	320	430
975	.550	.400	17	7	3.3	100	120	160	500	710	1.6k
975	.550	.400	16	4	3.2	75	96	120	160	210	290
983	.600	.400	16	5	3.3	81	89	110	180	240	420
982	.600	.400	14	3	3.2	110	160	230	480	820	2.3k
982	.600	.400	12	4	3.2	99	130	180	390	794	2.1k
980	.550	.400	18	5	3.3	85	100	160	220	-	N.O.
981	.550	.400	16	5	3.2	72	80	110	130	180	370
981	.550	.400	14	5	3.2	78	86	95	220	230	960
982	.550	.400	12	5	3.1	75	87	110	220	410	1.0k
983	.550	.400	10	5	3.1	84	100	120	270	450	1.1k
1000	.600	.400	16	5	3.4	77	94	160	110	184	490
1002	.600	.400	14	3	3.4	340	520	800	2.7k	-	N.O.
1000	.600	.400	12	2	3.3	150	180	230	790	-	N.O.
1000	.595	.400	16	4	3.4	110	120	140	200	230	270
1001	.595	.400	14	3	3.4	200	260	310	760	-	N.O.
1000	.590	.400	16	4	3.4	90	100	110	220	250	280
1006	.590	.400	14	2	3.4	150	180	250	2.2k	-	N.O.
1000	.590	.400	12	2	3.3	150	230	510	530	850	2.2k
1001	.585	.400	14	3	3.4	130	160	190	210	410	790
1000	.585	.400	12	2	3.3	300	420	700	1.4k	1.6k	1.8k

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1000	.580	.400	14	3	3.4	100	120	150	300	410	510
1004	.580	.400	12	2	3.3	220	230	240	1.2k	1.4k	1.6k
998	.550	.400	19	5	3.4	75	79	90	160	300	1.0k
999	.550	.350	18	5	2.6	79	88	98	170	210	260
999	.550	.400	18	6	3.4	73	83	98	100	190	360
1000	.550	.400	17	4	3.3	85	110	170	170	420	2.0k
1002	.550	.400	16	6	3.3	85	92	110	150	220	330
1000	.550	.400	15	5	3.2	83	93	110	140	180	290
1000	.550	.400	14	6	3.2	69	82	100	120	-	N.O.
1000	.550	.400	13	4	3.2	70	91	110	300	650	1.4k
1004	.550	.400	12	2	3.2	150	170	200	1.3k	1.5k	1.9k
999	.550	.400	11	6	3.1	78	130	180	170	-	N.O.
1001	.550	.400	10	2	3.3	600	690	800	3.0k	-	N.O.
999	.550	.400	8	5	3.1	960	1.3k	2.3k	1.5k	-	N.O.
999	.545	.400	18	2	3.4	91	99	109	200	240	300
1002	.545	.400	16	4	3.3	69	82	109	140	190	230
1000	.545	.400	14	3	3.2	78	84	89	130	160	190
1005	.500	.400	22	3	3.6	100	170	390	1.1k	-	N.O.
1002	.500	.350	22	4	2.8	100	120	160	230	-	N.O.
1006	.500	.400	20	3	3.5	140	160	190	280	370	560
1002	.500	.350	20	5	2.8	100	110	130	220	270	380
1003	.500	.400	18	5	3.4	120	150	190	160	370	920
1004	.500	1350	18	5	2.7	160	240	380	920	1.1k	1.7k

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	Co	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1004	.500	.400	16	2	3.3	280	320	400	720	-	N.O.
1005	.500	.350	16	3	2.7	380	440	550	1.2k	1.6k	2.0k
1009	.500	.400	14	2	3.2	350	500	880	800	-	N.O.
1008	.500	.350	14	3	2.6	590	790	1.2k	1.6k	-	N.O.
1020	.550	.400	18	5	3.5	98	110	140	250	580	2.5k
1021	.550	.400	16	5	3.4	91	100	110	180	300	500
1021	.550	.400	14	4	3.3	140	270	1.6k		N.O.	
1021	.550	.400	12	5	3.3	90	100	120	290	380	580
1022	.550	.400	10	5	3.2	120	160	270	620	880	1.3k
1026	.550	.400	6	6	3.3		N.O.			N.O.	
1040	.550	.400	18	5	3.6	90	100	120	190	220	270
1041	.550	.400	16	5	3.5	82	94	120	210	320	530
1041	.550	.400	14	5	3.2	160	200	280	410	810	2.0k
1042	.550	.400	12	5	3.4	140	190	280	540	690	1.8k
1042	.550	.400	10	2	3.4	1.7k	1.8k	1.8k		N.O.	
1044	.550	.400	8	4	3.3		N.O.			N.O.	
1051	.550	.400	18	4	3.6	80	81	110	160	280	920
1050	.550	.400	17	6	3.5	62	79	90	110	160	180
1051	.550	.400	16	5	3.4	70	83	100	90	150	250
1050	.550	.400	15	5	3.5	75	86	110	140	200	300
1051	.550	.400	14	2	3.4	1.3k	1.6k	2.2k		N.O.	

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resisgance(055 Minimum Low	Mean	High	Maximum Low	Mean	High
1082	.550	.400	16	4	3.6	83	110	130	350	460	610
1081	.550	.400	14	3	3.5	340	440	650	750	-	N.O.
1080	.550	.400	12	5	3.5	210	330	1.2k		N.O.	
1081	.550	.400	10	5	3.4	180	290	390	1.6k	-	N.O.
1083	.550	.400	8	2	3.3	800	850	900	2.5k	-	N.O.
1101	.550	.400	17	6	3.6	180	250	360	590	860	2.0k
1100	.550	.400	16	5	3.6	230	380	1.9k	2.8k	-	N.O.
1104	.550	.400	15	2	3.5	470	640	1.0k	3.0k	-	N.O.
1101	.550	.400	14	8	3.4	130	210	450	360	-	N.O.
1101	.550	.400	13	5	3.4	130	210	310	870	-	N.O.
1103	.550	.400	12	7	3.4	130	230	1.4k	300	-	N.O.
1103	.550	.400	11	2	3.4	500	560	640		N.O.	
1101	.550	.400	10	4	3.3	400	650	1.1k		N.O.	
1101	.500	.400	17	5	3.7	70	93	140	180	230	260
1100	.500	.400	16	5	3.7	82	97	140	140	180	270
1102	.500	.400	15	5	3.6	77	110	230	160	-	N.O.
1100	.500	.400	14	5	3.6	72	90	120	150	230	390
1100	.500	.400	13	4	3.6	98	160	330	170	340	2.0k
1102	.500	.400	12	4	3.5	94	120	160	200	310	780
1104	.500	1400	11	3	3.4	160	240	370	440	560	680
1106	.500	.400	10	3	3.4	120	190	400	770	1.5k	3.0k

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55^o to $+90^oC$) Minimum Low	Mean	High	Maximum Low	Mean	High
1126	.550	.400	16	5	3.7	120	200	350	440	1.1k	2.0k
1126	.550	.400	14	5	3.6	110	130	200	310	380	440
1126	.550	.400	12	5	3.5	140	180	220	490	780	1.6k
1124	.550	.400	10	4	3.6	480	720	1.2k	1.5k	1.9k	2.3k
1126	.550	.400	8	9	3.4		N.O.			N.O.	
1201	.550	.400	15	4	3.9	63	90	140	180	220	280
1200	.550	.400	14	3	3.8	95	100	110	200	230	300
1202	.550	.400	13	4	3.8	88	100	120	180	250	260
1202	.550	.400	12	3	3.8	100	130	200	380	-	N.O.
1202	.550	.400	11	5	3.8	65	76	95	120	120	140
1203	.550	.400	10	5	3.7	120	160	230	220	640	5.0k
1200	.550	.400	9	5	3.6	78	110	180	140	280	390
1202	.550	.400	8	5	3.6	190	310	450	980	1.5k	2.4k
1203	.500	.400	18	3	4.2	350	470	580	1.0k	1.2k	1.8k
1203	.500	.400	16	6	4.0		N.O.			N.O.	
1202	.500	.400	14	6	3.9		N.O.			N.O.	
1203	.500	.400	12	3	3.8	600	700	1.0k	1.0k	2.2k	6.6k
1203	.500	.400	10	5	3.7	380	630	2.5k	920	-	N.O.
1226	.550	.400	15	4	4.0	110	130	150	220	340	480
1226	.550	.400	13	5	3.8	160	200	390	600	780	1.0k
1226	.550	.400	11	5	3.8	180	260	380	800	1.4k	2.7k
1226	.550	.400	9	5	3.7	230	450	1.5k	1.4k	-	N.O.

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1250	.550	.400	14	4	4.0	240	460	850	1.8k	-	N.O.
1251	.550	.400	12	5	3.9	160	290	800	300	-	N.O.
1252	.550	.400	10	5	3.6	180	280	1.3k	540	-	N.O.
1249	.550	.400	8	4	3.7	160	260	470	720	-	N.O.
1252	.550	.400	6	4	3.5	350	760	2.3k	920	-	N.O.
1301	.550	.400	14	4	4.3	120	160	220	840	1.0k	1.2k
1301	.550	.400	12	5	4.2	52	59	70	430	720	1.4k
1303	.550	.400	10	4	4.0	53	62	85	180	200	220
1301	.550	.400	8	6	3.9	100	170	240	400	570	1.0k
1301	.550	.400	6	4	3.8	120	170	320	560	780	1.5k
1326	.550	.400	14	4	4.2	55	79	120	680	1.2k	3.0k
1326	.550	.400	12	5	4.1	50	56	70	220	390	620
1327	.550	.400	10	5	4.0	52	62	77	140	170	200
1326	.550	.400	8	5	3.9	53	60	72	110	130	140
1328	.550	.400	6	4	3.8	120	240	1.6k	1.0k	-	N.O.
1401	.550	.400	13	8	4.5	43	49	68	83	120	170
1400	.550	.400	12	6	4.4	33	46	65	150	170	200
1401	.550	.400	11	5	4.3	48	53	61	120	160	180
1401	.550	.400	10	4	4.3	67	84	110	160	230	400
1398	.550	.400	9	4	4.1	65	73	82	100	160	340
1400	.550	.400	8	5	4.0	68	70	75	140	410	1.5k
1400	.550	.400	7	5	4.0	120	220	720	240	630	1.9k

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1512	.550	.400	12	5	4.8	52	60	65	92	110	130
1502	.550	.400	10	4	4.7	50	56	68	140	150	160
1498	.550	.400	8	4	4.6	95	100	120	180	-	N.O.
1496	.550	.400	6	4	4.4	180	200	270	300	390	620
1600	.550	.400	11	3	4.9	73	76	80	140	160	210
1603	.550	.400	10	4	4.9	85	160	240	380	520	800
1599	.550	.400	9	4	4.7	90	140	230	320	410	700
1606	.550	.400	8	2	4.6	100	110	130	230	260	310
1601	.550	.400	7	5	4.5	63	82	110	210	230	260
1606	.550	.400	6	2	4.5	65	77	90	100	170	240
1804	.550	.400	9	5	5.3	35	38	46	79	100	120
1806	.550	.400	8	3	5.3	48	50	54	100	140	330
1805	.550	.400	7	4	5.1	36	38	39	280	340	480
1803	.550	.400	6	5	5.0	35	45	61	98	130	160
1802	.550	.400	5	5	4.9	43	52	65	86	130	130
1999	.550	.400	8	3	6.3	50	52	55	92	130	200
2002	.550	.310	8	9	3.7	33	38	45	120	170	220
1996	.550	.400	7	4	5.9	50	54	70	230	240	260
2003	.550	.400	6	5	5.8	55	61	69	180	200	240
1999	.550	.400	5	5	5.6	59	65	80	200	220	260
2004	.550	.310	4	10	3.5	55	66	83	160	180	210

TABLE 5 cont'd.

BI-CONVEX

Freq KC	Plate Dia	Elec Dia	Curv Diop	No.of Units	C_o	Series Resistance(-55° to +90°C) Minimum Low	Mean	High	Maximum Low	Mean	High
1997	.550	.310	2	5	3.5	92	120	170	160	220	300
1998	.550	.310	1	4	3.5	250	500	1.5k	480	930	2.5k
2499	.550	.400	7	4	7.5	63	69	75	130	140	160
2508	.550	.400	6	4	7.7	36	40	49	130	140	150
2502	.550	.400	5	4	7.2	45	46	48	150	170	210
2501	.550	.400	4	5	6.9	45	46	48	100	110	120
2505	.550	.400	3	5	6.6	28	29	32	45	46	51
3010	.550	.350	5.0	5	6.3	45	63	85	180	200	240
3014	.550	.350	4.0	5	6.3	29	34	40	140	160	180
3015	.550	.350	3.0	5	6.2	15	17	19	23	25	26
3012	.550	.350	2.5	5	5.8	14	15	17	16	19	25
3005	.550	.350	2.0	5	5.8	10	11	14	15	16	20
3002	.550	.350	1.5	5	5.7	10	13	18	20	42	110
3010	.550	.350	1.0	5	5.6	12	16	20	18	36	75
4006	.550	.310	2.5	5	6.1	12	13	13	20	22	24
4006	.550	.310	2.0	5	6.0	11	12	13	19	21	25
4000	.550	.310	1.5	5	6.2	14	16	19	86	87	93
3999	.550	.310	1.0	5	5.8	7.0	7.9	9.0	8.3	9.2	10

APPENDIX A

PARASITIC MODE DATA

The discussion of the parasitic modes of vibration of piezoelectric resonators to be found in the literature is extensive, but by no means comprehensive. The subject has been approached both theoretically and experimentally by various investigators for a number of years, but the extreme complexity of the problem makes a full treatment exceedingly difficult. The data offered here is not the result of any extensive investigation of the subject, but rather information obtained in the course of the main program.

Figure 33 is a graph of frequency-thickness coefficient versus diameter to thickness ratio for the thickness-shear mode and three parasitic modes, for plane-parallel, circular, AT plates. These curves were computed, using constants derived from experimental measurements, neglecting the coupling between modes. The values for the flexure mode coefficients were extrapolated from data published by Lowell Technological Institute.* Only those overtone orders (n) that are likely to exhibit strong coupling to the thickness-shear mode have been plotted.

The parasitic mode represented by the series of points in Figure 34 has not been definitely identified. This mode is unique in that there appears to be virtually no "pulling" of the main mode frequency associated with it. Its affect on the resistance-temperature characteristics can be seen in the upper curve of Figure 2. The two large dips near 90° are caused by this mode. Its temperature coefficient of frequency is approximately ten times that of the parasitic modes causing the other dips in this curve, namely about 150 parts per million per degree centigrade.

* Research Foundation, Lowell Technological Institute, Final Report, DA 36-039-sc-72833, 1957

FIGURE 33

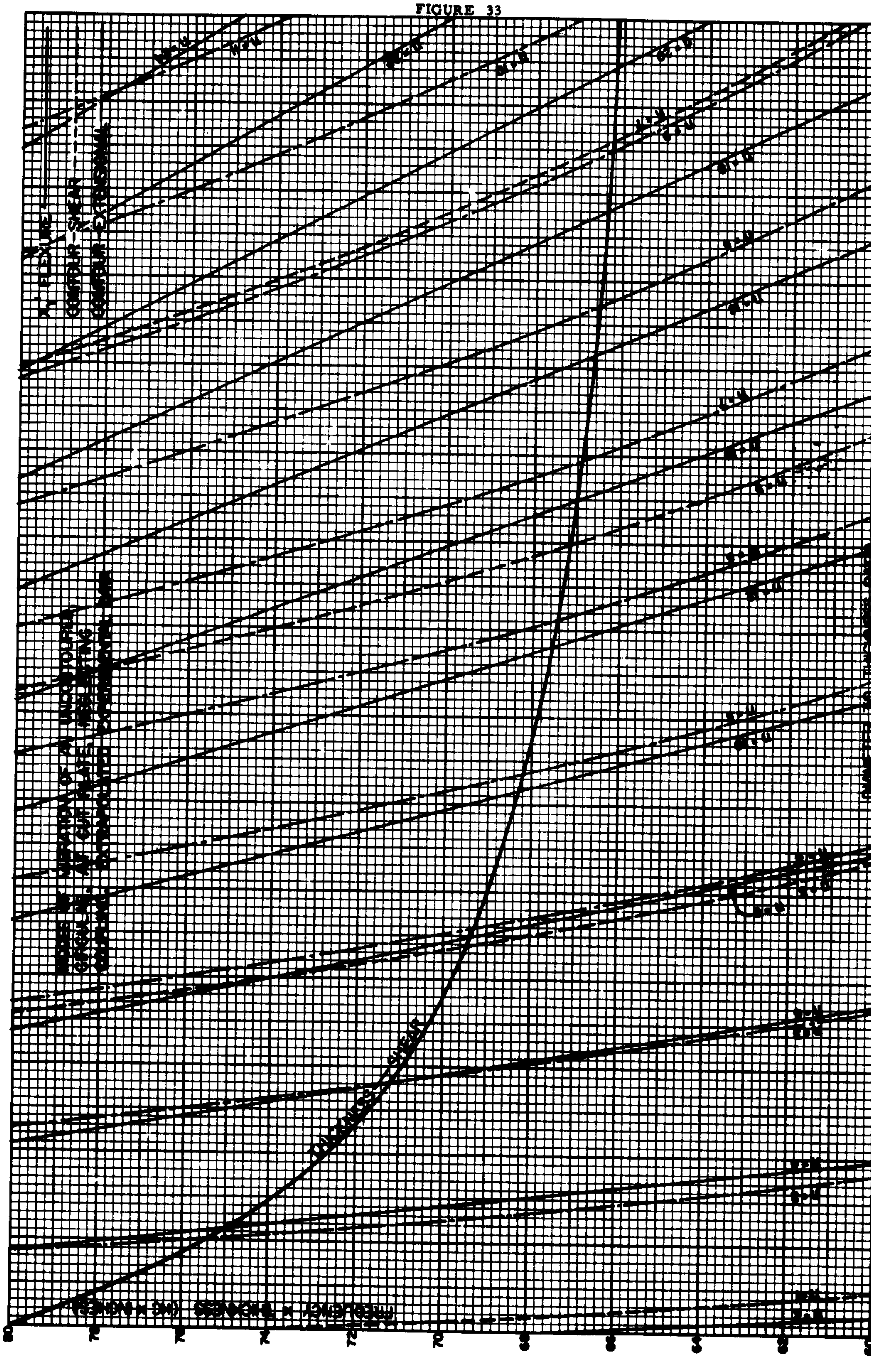
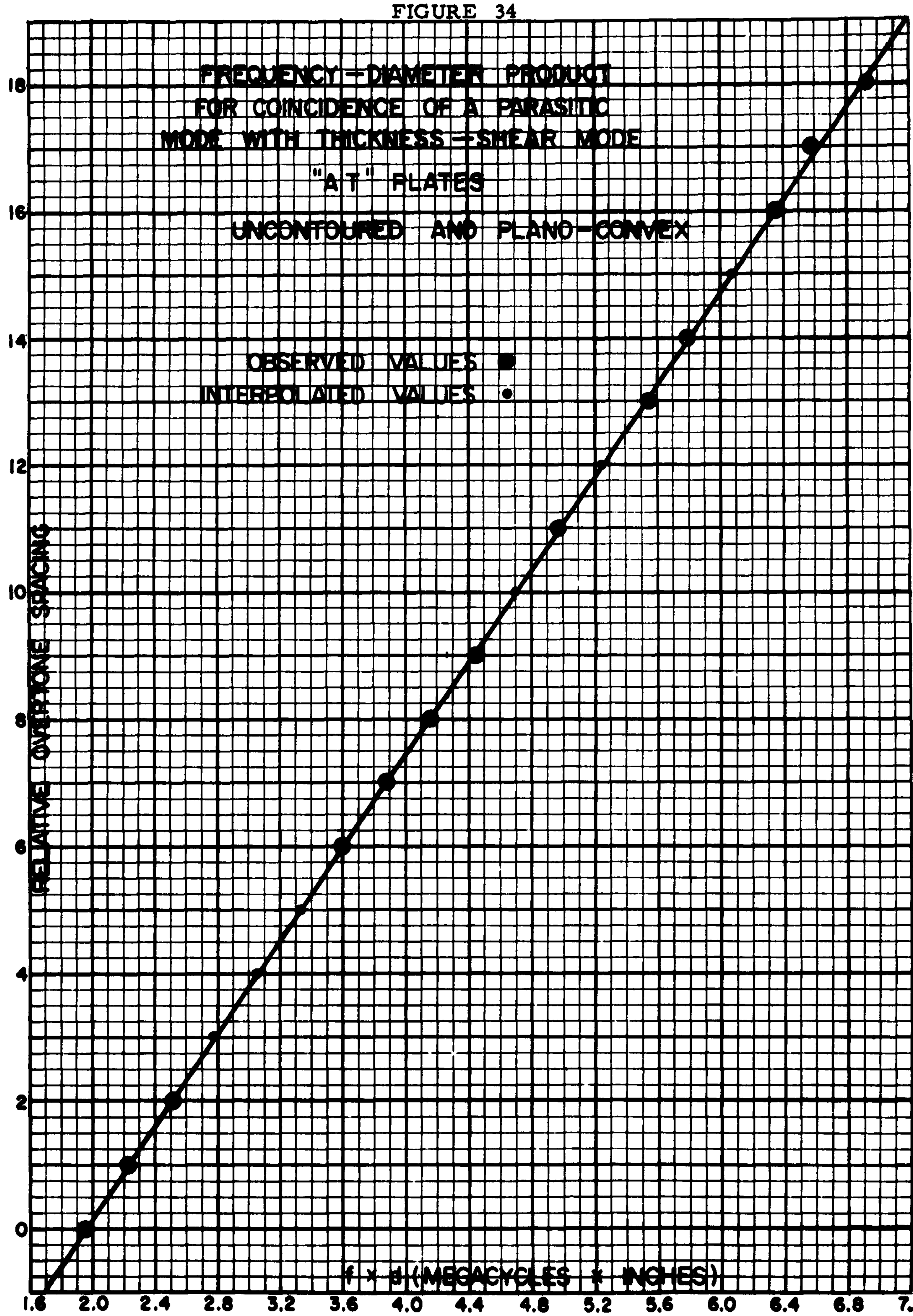

FLEXURE
CONTOUR-SHEAR
CONTOUR-EXTENSIONAL
THICKNESS-SHEAR
DIAMETER TO THICKNESS RATIO

FIGURE 34


FREQUENCY — DIAMETER PRODUCT
FOR COINCIDENCE OF A PARASITIC
MODE WITH THICKNESS — SHEAR MODE
"AT" PLATES
UNCONTOURED AND PLANO — CONVEX
OBSERVED VALUES
INTERPOLATED VALUES
RELATIVE OVERTONE SPACING
f × d (MEGACYCLES × INCHES)
18
16
14
12
10
8
6
4
2
0
1.6
2.0
2.4
2.8
3.2
3.6
4.0
4.4
4.8
5.2
5.6
6.0
6.4
6.8
7.2

This mode has been observed with both uncontoured and plano-convex plates, apparently without being appreciably affected by the curvature of the surface. The overtone spacing shown in Figure 34 is approximately equal to the value to be expected for the contour-shear mode, if we assume that only odd or even overtone orders are observed. A possible explanation for the fact that the dips usually occur in pairs, if it actually is the contour-shear mode, is the slight difference in the frequency coefficients associated with the X and Z' dimensions that has been observed for this mode in rectangular plates.

APPENDIX B

FREQUENCY-THICKNESS DATA

The frequency-thickness coefficient of an AT type resonator is a function of the configuration of the plate. For circular, plane-parallel plates, it can be expressed in terms of the diameter to thickness ratio. Measurements of frequency versus thickness were made with plates ranging in diameter to thickness ratio from 1 to 100. A graph of these results is shown in Figure 35. The values for the coefficient for very small d/t ratios should not be regarded as exact, because the very strong coupling between the thickness-shear mode and various parasitic modes of vibration makes it difficult to obtain reliable frequency measurements. The value of 65.2 kilocycle-inches for the frequency-thickness coefficient of an "infinite" plate appears to be the best available figure.

The frequency-thickness coefficient for spherically contoured plates is a function of the curvature of the surface as well as the diameter to thickness ratio. If the curvature is great enough to make the thickness at the edge of the plate much less than the thickness in the center, the actual influence of the plate diameter becomes small, compared to the influence of the curvature. Under these conditions, the frequency-thickness coefficient can be expressed as a function of the "relative curvature" (the ratio of the plate thickness to the radius of curvature). Empirical formulas have been derived to fit the experimental measurements made on fully contoured plates, both plano-convex and bi-convex. The data listed in Tables 6 and 7 was computed from these formulas. The frequency-thickness coefficient is given in kilocycle-inches, the plate thickness in thousandths of an inch, and the relative curvature is multiplied by a factor of 1000.

FIGURE 35

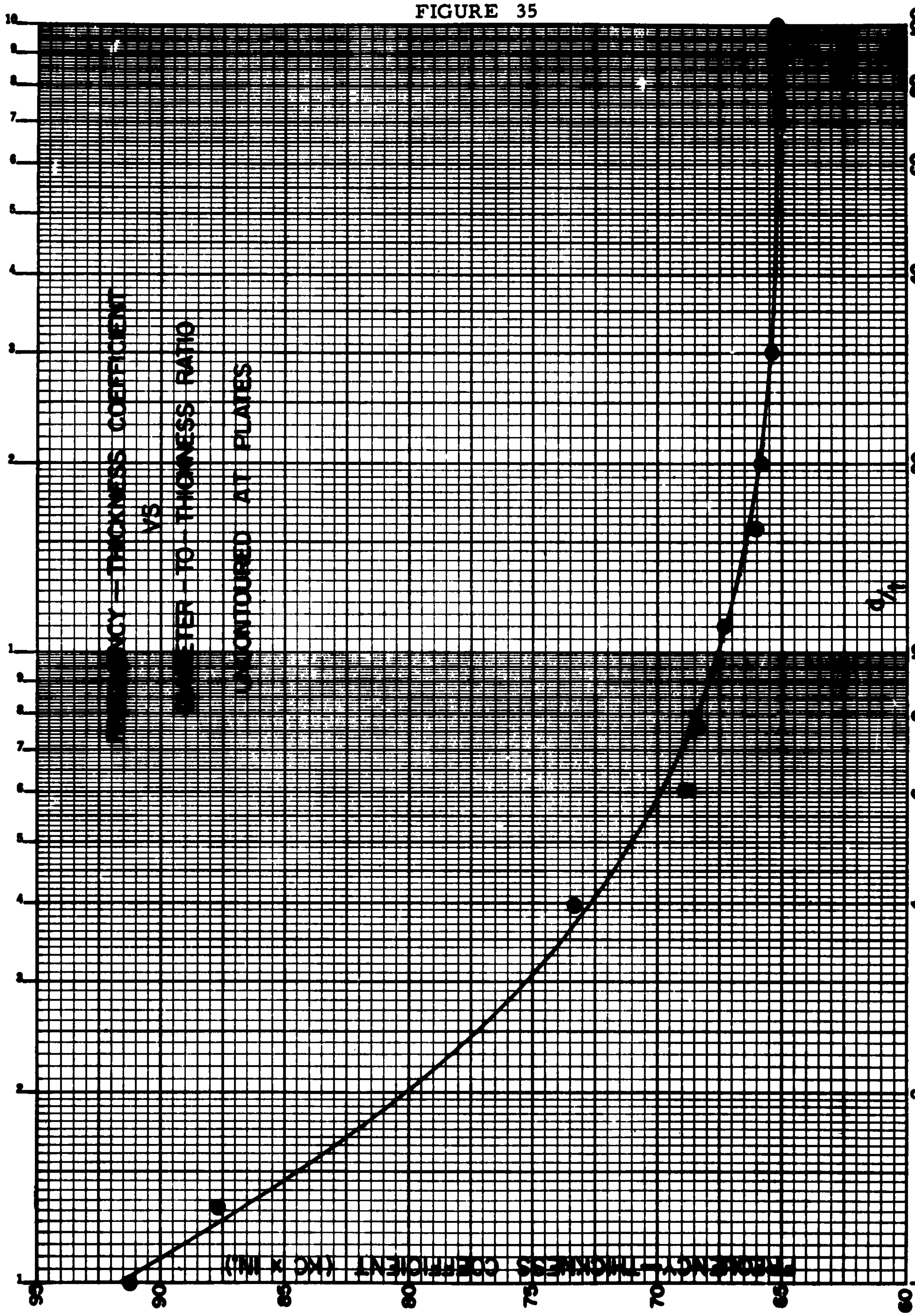


TABLE 6

PLANO - CONVEX									
Freq. KC	2 Diopters			4 Diopters			6 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1000	67.9	67.9	6.53	69.0	69.0	13.2	69.8	69.8	20.1
1100	67.7	61.6	5.90	68.8	62.5	12.0	69.6	63.3	18.2
1200	67.6	56.4	5.40	68.6	57.2	11.0	69.4	57.8	16.6
1300	67.5	52.0	4.98	68.5	52.7	10.1	69.2	53.3	15.3
1400	67.4	48.2	4.62	68.4	48.8	9.36	69.1	49.3	14.2
1500	67.4	44.9	4.30	68.3	45.5	8.73	69.0	46.0	13.2
1600	67.3	42.1	4.03	68.2	42.6	8.17	68.8	43.0	12.4
1700	67.2	39.6	3.79	68.1	40.1	7.68	68.7	40.4	11.6
1800	67.2	37.3	3.58	68.0	37.8	7.24	68.6	38.1	11.0
1900	67.1	35.3	3.39	67.9	35.8	6.85	68.5	36.1	10.4
2000	67.1	33.5	3.22	67.9	33.9	6.49	68.4	34.2	9.84
2100	67.0	31.9	3.06	67.8	32.3	6.19	68.4	32.6	9.36
2200	67.0	30.4	2.92	67.7	30.8	5.90	68.3	31.0	8.93
2300	67.0	29.1	2.79	67.7	29.4	5.64	68.2	29.7	8.53
2400	66.9	27.9	2.82	67.6	28.2	5.40	68.2	28.4	8.17
2500	66.9	26.8	2.56	67.6	27.0	5.18	68.1	27.2	7.84
2600	66.8	25.7	2.46	67.5	26.0	4.98	68.0	26.2	7.53
2700	66.8	24.8	2.37	67.5	25.0	4.79	68.0	25.2	7.24
2800	66.8	23.8	2.28	67.4	24.1	4.62	68.0	24.3	6.98
2900	66.8	23.0	2.21	67.4	22.9	4.46	67.9	23.4	6.73
3000	66.7	22.2	2.13	67.4	22.5	4.30	67.9	22.6	6.50

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	8 Diopters			10 Diopters			12 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1000	70.5	70.5	27.0	71.1	71.1	34.1	71.7	71.7	41.2
1100	70.3	63.9	24.5	70.9	64.4	30.9	71.4	64.9	37.3
1200	70.1	58.4	22.4	70.6	58.8	28.2	71.1	59.3	34.1
1300	69.9	53.7	20.6	70.4	54.2	26.0	71.0	54.5	31.4
1400	69.7	49.8	19.1	70.2	50.2	24.0	70.7	50.5	29.1
1500	69.5	46.4	17.8	70.0	46.7	22.4	70.5	47.0	27.0
1600	69.4	43.4	16.6	69.9	43.7	20.9	70.3	44.0	25.3
1700	69.3	40.8	15.6	69.8	41.0	19.7	70.2	41.3	23.8
1800	69.2	38.4	14.7	69.6	38.7	18.5	70.0	38.9	22.4
1900	69.1	36.3	13.9	69.5	36.6	17.5	69.9	36.8	21.2
2000	69.0	34.5	13.2	69.4	34.7	16.6	69.8	34.9	20.1
2100	68.9	32.8	12.6	69.3	33.0	15.8	69.7	33.2	19.1
2200	68.9	31.3	12.0	69.2	31.4	15.1	69.6	31.6	18.2
2300	68.7	29.9	11.5	69.1	30.0	14.4	69.5	30.2	17.4
2400	68.6	28.6	11.0	69.0	28.8	13.8	69.4	28.9	16.6
2500	68.6	27.4	10.5	69.0	27.6	13.2	69.3	27.7	16.0
2600	68.5	26.3	10.1	68.9	26.5	12.7	69.2	26.6	15.3
2700	68.4	25.3	9.72	68.8	25.5	12.2	69.2	25.6	14.7
2800	68.4	24.4	9.35	68.8	24.6	11.8	69.1	24.7	14.2
2900	68.3	23.6	9.03	68.7	23.7	11.4	69.0	23.8	13.7
3000	68.3	22.8	8.73	68.6	22.9	11.0	69.0	23.0	13.2

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	14 Diopters			16 Diopters			18 Diopters		
	ft	t	t/r	ft	t	t/R	ft	t	t/R
1000	72.2	72.2	48.5	72.7	72.7	55.8	73.2	73.2	63.1
1100	71.9	65.4	43.9	72.4	65.8	50.4	72.8	66.2	57.1
1200	71.6	59.7	40.0	72.1	60.0	46.0	72.5	60.4	52.1
1300	71.4	54.9	36.8	71.8	55.2	42.4	72.2	55.3	47.9
1400	71.1	50.8	34.1	71.6	51.1	39.2	71.9	51.4	44.3
1500	70.9	47.3	31.7	71.3	47.6	36.5	71.7	47.8	41.2
1600	70.8	44.2	29.7	71.1	44.5	34.1	71.5	44.7	38.6
1700	70.6	41.5	27.9	71.0	41.7	32.0	71.3	42.0	36.2
1800	70.4	39.1	26.3	70.8	39.3	30.2	71.1	39.5	34.1
1900	70.3	37.0	24.8	70.6	37.2	28.5	71.0	37.4	32.2
2000	70.2	35.1	23.5	70.5	35.3	27.0	70.8	35.4	30.6
2100	70.0	33.4	22.4	70.4	33.5	25.7	70.7	33.7	29.0
2200	69.9	31.8	21.2	70.3	31.9	24.5	70.6	32.1	27.7
2300	69.8	30.4	20.4	70.2	30.5	23.4	70.5	30.6	26.4
2400	69.7	29.1	19.5	70.1	29.2	22.4	70.3	29.3	25.3
2500	69.6	27.9	18.7	70.0	28.0	21.5	70.2	28.1	24.2
2600	69.6	26.8	18.0	69.9	26.9	20.6	70.1	27.0	23.3
2700	69.5	25.7	17.3	69.8	25.8	19.8	70.0	25.9	22.4
2800	69.4	24.8	16.6	69.7	24.9	19.1	70.0	25.0	21.6
2900	69.3	23.9	16.0	69.6	24.0	18.4	69.9	24.1	20.8
3000	69.3	23.1	15.5	69.5	23.2	17.8	69.8	23.3	20.1

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	20 Diopters			22 Diopters			24 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1000	73.6	73.6	70.6	74.0	74.0	78.0	74.4	74.4	85.6
1100	73.2	66.6	63.8	73.6	66.9	70.6	74.0	67.2	77.4
1200	72.9	60.7	58.2	73.2	61.0	64.3	73.6	61.3	70.6
1300	72.6	55.8	53.5	72.9	56.1	59.1	73.3	56.4	64.8
1400	72.3	51.6	49.5	72.6	51.9	54.7	73.0	52.1	60.0
1500	72.1	48.0	46.0	72.4	48.3	50.9	72.7	48.5	55.8
1600	71.8	44.9	43.0	72.2	45.1	47.6	72.5	45.3	52.1
1700	71.6	42.1	40.4	72.0	42.3	44.6	72.3	42.5	48.9
1800	71.5	39.7	38.1	71.8	39.9	42.0	72.1	40.0	46.0
1900	71.3	37.5	36.0	71.6	37.7	39.7	71.9	37.8	43.5
2000	71.1	35.6	34.1	71.4	35.7	37.7	71.7	35.8	41.2
2100	71.0	33.8	32.4	71.3	33.9	35.8	71.6	34.1	39.2
2200	70.9	32.2	30.9	71.1	32.3	34.1	71.4	32.4	37.3
2300	70.7	30.8	29.5	71.0	30.9	32.6	71.3	31.0	35.6
2400	70.6	29.4	28.2	70.9	29.5	31.2	71.1	29.6	34.1
2500	70.5	28.2	27.0	70.8	28.3	29.8	71.0	28.4	32.7
2600	70.4	27.1	26.0	70.7	27.2	28.7	70.9	27.3	31.4
2700	70.3	26.0	25.0	70.6	26.1	27.6	70.8	26.2	30.2
2800	70.2	25.1	24.0	70.5	25.2	26.5	70.7	25.2	29.0
2900	70.1	24.2	23.2	70.4	24.3	25.6	70.6	24.3	28.0
3000	70.0	23.4	22.4	70.4	23.5	24.7	70.5	23.5	27.0

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	1 Diopter ft	1 Diopter t	1 Diopter t/R	2 Diopters ft	2 Diopters t	2 Diopters t/R	3 Diopters ft	3 Diopters t	3 Diopters t/R
3000	66.3	22.1	1.06	66.7	22.2	2.13	67.1	22.4	3.22
3200	66.2	20.7	.992	66.7	20.8	2.00	67.0	20.9	3.01
3400	66.2	19.5	.934	66.6	19.6	1.88	67.0	19.7	2.83
3600	66.2	18.4	.881	66.6	18.5	1.77	66.9	18.6	2.67
3800	66.2	17.4	.835	66.6	17.5	1.68	66.9	17.6	2.53
4000	66.1	16.5	.792	66.5	16.6	1.59	66.8	16.7	2.40
4200	66.1	15.7	.755	66.5	15.8	1.52	66.8	15.9	2.29
4400	66.1	15.0	.720	66.5	15.1	1.45	66.8	15.2	2.18
4600	66.1	14.4	.688	66.4	14.4	1.38	66.7	14.5	2.08
4800	66.1	13.8	.660	66.4	13.8	1.33	66.7	13.9	2.00
5000	66.0	13.2	.633	66.4	13.3	1.27	66.7	13.3	1.92
5500	66.0	12.0	.575	66.3	12.1	1.16	66.6	12.1	1.74
6000	66.0	11.0	.527	66.3	11.0	1.06	66.5	11.1	1.59
6500	65.9	10.1	.486	66.2	10.2	.977	66.5	10.2	1.47
7000	65.9	9.42	.452	66.2	9.46	.906	66.4	9.49	1.36
7500	65.9	8.78	.421	66.2	8.82	.846	66.4	8.85	1.27
8000	65.9	8.23	.395	66.1	8.27	.792	66.4	8.29	1.19
8500	65.8	7.75	.371	66.1	7.78	.746	66.3	7.80	1.12
9000	65.8	7.31	.350	66.1	7.34	.704	66.3	7.36	1.06
9500	65.8	6.93	.332	66.1	6.95	.666	66.3	6.98	1.00
10000	65.8	6.58	.315	66.0	6.60	.633	66.2	6.62	.952

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	4 Diopters			5 Diopters			6 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
3000	67.4	22.5	4.30	67.6	22.5	5.40	67.9	22.6	6.50
3200	67.3	21.0	4.03	67.6	21.1	5.06	67.8	21.2	6.09
3400	67.2	19.8	3.79	67.5	19.8	4.76	67.7	19.9	5.73
3600	67.2	18.7	3.58	67.4	18.7	4.49	67.6	18.8	5.40
3800	67.1	17.7	3.39	67.4	17.7	4.25	67.6	17.8	5.11
4000	67.1	16.8	3.22	67.3	16.8	4.03	67.5	16.9	4.85
4200	67.0	16.0	3.06	67.2	16.0	3.84	67.4	16.1	4.62
4400	67.0	15.2	2.92	67.2	15.3	3.66	67.4	15.3	4.40
4600	67.0	14.6	2.79	67.2	14.6	3.50	67.3	14.6	4.21
4800	66.9	13.9	2.67	67.1	14.0	3.35	67.3	14.0	4.03
5000	66.9	13.4	2.56	67.1	13.4	3.22	67.3	13.4	3.87
5500	66.8	12.1	2.33	67.0	12.2	2.92	67.2	12.2	3.51
6000	66.7	11.1	2.13	66.9	11.2	2.67	67.1	11.2	3.22
6500	66.7	10.3	1.97	66.8	10.3	2.47	67.0	10.3	2.96
7000	66.6	9.52	1.82	66.8	9.54	2.29	66.9	9.56	2.75
7500	66.6	8.88	1.70	66.7	8.90	2.13	66.9	8.92	2.56
8000	66.5	8.32	1.59	66.7	8.34	2.00	66.8	8.35	2.40
8500	66.5	7.82	1.50	66.6	7.84	1.88	66.7	7.84	2.26
9000	66.4	7.38	1.42	66.6	7.40	1.77	66.7	7.41	2.13
9500	66.4	6.99	1.34	66.6	7.01	1.68	66.7	7.02	2.02
10000	66.4	6.64	1.27	66.5	6.65	1.59	66.7	6.67	1.92

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	7 Diopters			8 Diopters			9 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
3000	68.1	22.7	7.61	68.3	22.8	8.73	68.4	22.8	9.84
3200	68.0	21.2	7.13	68.2	21.3	8.17	68.4	21.4	9.21
3400	67.9	20.0	6.70	68.1	20.0	7.68	68.3	20.1	8.66
3600	67.8	18.8	6.32	68.0	18.9	7.23	68.2	18.9	8.17
3800	67.8	17.8	5.98	67.9	17.9	6.85	68.1	17.9	7.73
4000	67.7	16.9	5.68	67.9	17.0	6.50	68.0	17.0	7.34
4200	67.6	16.1	5.40	67.8	16.1	6.19	68.0	16.2	6.98
4400	67.6	15.4	5.15	67.7	15.4	5.90	67.9	15.4	6.65
4600	67.5	14.7	4.92	67.7	14.7	5.64	67.8	14.8	6.36
4800	67.5	14.1	4.71	67.6	14.1	5.40	67.8	14.1	6.09
5000	67.4	13.5	4.52	67.6	13.5	5.18	67.7	13.5	5.84
5500	67.3	12.2	4.11	67.5	12.3	4.71	67.6	12.3	5.30
6000	67.2	11.2	3.76	67.4	11.2	4.30	67.5	11.2	4.85
6500	67.2	10.3	3.47	67.3	10.4	3.97	67.4	10.4	4.48
7000	67.1	9.58	3.22	67.2	9.60	3.68	67.3	9.62	4.15
7500	67.0	8.94	3.00	67.1	8.95	3.43	67.3	8.97	3.87
8000	67.0	8.37	2.81	67.1	8.38	3.22	67.2	8.40	3.62
8500	66.9	7.87	2.64	67.0	7.88	3.02	67.1	7.90	3.41
9000	66.9	7.43	2.49	67.0	7.44	2.85	67.1	7.45	3.22
9500	66.8	7.03	2.36	66.9	7.04	2.70	67.0	7.06	3.04
10000	66.8	6.68	2.24	66.9	6.69	2.56	67.0	6.70	2.89

TABLE 6 cont'd.

PLANO - CONVEX									
Freq. KC	10 Diopters			11 Diopters			12 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
3000	68.6	22.9	11.0	68.8	22.9	12.1	69.0	23.0	13.2
3200	68.5	21.4	10.3	68.7	21.5	11.3	68.8	21.5	12.4
3400	68.4	20.1	9.64	68.6	20.2	10.6	68.7	20.2	11.6
3600	68.3	19.0	9.10	68.5	19.0	10.0	68.6	19.1	11.0
3800	68.2	18.0	8.61	68.4	18.0	9.49	68.5	18.0	10.4
4000	68.2	17.0	8.17	68.3	17.1	9.00	68.4	17.1	9.84
4200	68.1	16.2	7.77	68.2	16.2	8.56	68.4	16.3	9.36
4400	68.0	15.5	7.41	68.2	15.5	8.17	68.3	15.5	8.90
4600	68.0	14.8	7.08	68.1	14.8	7.81	68.2	14.8	8.53
4800	67.9	14.2	6.78	68.0	14.2	7.47	68.2	14.2	8.17
5000	67.9	13.6	6.50	68.0	13.6	7.17	68.1	13.6	7.83
5500	67.7	12.3	5.90	67.9	12.3	6.50	68.0	12.4	7.11
6000	67.6	11.3	5.40	67.7	11.3	5.95	67.9	11.3	6.50
6500	67.5	10.4	4.98	67.6	10.4	5.49	67.8	10.4	6.00
7000	67.4	9.63	4.62	67.6	9.65	5.09	67.7	9.67	5.56
7500	67.4	8.98	4.30	67.5	9.00	4.74	67.6	9.01	5.18
8000	67.3	8.41	4.03	67.4	8.42	4.44	67.5	8.44	4.85
8500	67.2	7.91	3.79	67.3	7.92	4.18	67.4	7.93	4.56
9000	67.2	7.46	3.58	67.3	7.48	3.94	67.4	7.49	4.30
9500	67.1	7.07	3.39	67.2	7.08	3.73	67.3	7.08	4.08
10000	67.1	6.71	3.22	67.2	6.72	3.54	67.3	6.73	3.87

TABLE 7

BI - CONVEX									
Freq. KC	2 Diopters			4 Diopters			6 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1200	68.5	57.1	5.47	69.9	58.2	11.2	70.9	59.1	17.0
1250	68.4	54.7	5.25	69.8	55.8	10.7	70.8	56.6	16.3
1300	68.4	52.6	5.03	69.7	53.6	10.3	70.7	54.4	15.6
1350	68.3	50.6	4.85	69.6	51.6	9.88	70.6	52.3	15.0
1400	68.2	48.8	4.67	69.5	49.7	9.52	70.5	50.4	14.5
1450	68.2	47.0	4.51	69.4	47.9	9.18	70.4	48.6	14.0
1500	68.2	45.4	4.35	69.4	46.2	8.87	70.3	46.9	13.5
1550	68.1	43.9	4.21	69.3	44.7	8.57	70.2	45.3	13.0
1600	68.1	42.5	4.08	69.2	43.3	8.30	70.2	43.8	12.6
1650	68.0	41.2	3.95	69.2	41.9	8.03	70.1	42.5	12.2
1700	68.0	40.0	3.83	69.1	40.7	7.80	70.0	41.2	11.8
1750	67.9	38.8	3.72	69.1	39.5	7.57	69.9	40.0	11.5
1800	67.9	37.7	3.62	69.0	38.3	7.35	69.9	38.8	11.2
1850	67.9	36.7	3.52	69.0	37.3	7.15	69.8	37.7	10.8
1900	67.8	35.7	3.42	68.9	36.3	6.95	69.7	36.7	10.6
1950	67.8	34.8	3.33	68.9	35.3	6.77	69.7	35.7	10.3
2000	67.8	33.9	3.25	68.8	34.4	6.60	69.6	34.8	10.0
2100	67.7	32.2	3.09	68.7	32.7	6.28	69.5	33.1	9.52
2200	67.6	30.07	2.95	68.6	31.2	5.98	69.4	31.6	9.07
2300	67.6	29.4	2.82	68.6	29.8	5.72	69.3	30.1	8.67
2400	67.5	28.1	2.70	68.5	28.5	5.47	69.2	28.8	8.30
2500	67.5	27.0	2.59	68.4	27.4	5.25	69.2	27.7	7.96

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	8 Diopters			10 Diopters			12 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1200	71.8	59.8	22.9	72.6	60.5	28.9	73.3	61.1	35.1
1250	71.7	57.3	22.0	72.4	57.9	27.8	73.1	58.5	33.6
1300	71.5	55.0	21.1	72.3	55.6	26.6	73.0	56.1	32.3
1350	71.4	52.9	20.3	72.2	53.4	25.6	72.8	53.9	31.0
1400	71.3	50.9	19.5	72.0	51.4	24.7	72.7	51.9	30.0
1450	71.2	49.1	18.8	71.9	49.6	23.8	72.6	50.0	28.8
1500	71.1	47.4	18.2	71.8	47.9	22.9	72.4	48.3	27.8
1550	71.0	45.8	17.6	71.7	46.2	22.2	72.3	46.6	26.8
1600	70.9	44.3	17.0	71.6	44.7	21.4	72.2	45.1	26.0
1650	70.8	42.9	16.5	71.5	43.3	20.8	72.1	43.7	25.1
1700	70.7	41.6	16.0	71.4	42.0	20.1	72.0	42.3	24.4
1750	70.7	40.4	15.5	71.3	40.8	19.5	71.9	41.1	23.6
1800	70.6	39.2	15.0	71.2	39.6	19.0	71.8	39.9	22.9
1850	70.5	38.1	14.6	71.1	38.4	18.4	71.7	38.8	22.3
1900	70.4	37.1	14.2	71.1	37.4	17.9	71.6	37.7	21.7
1950	70.4	36.1	13.8	71.0	36.4	17.4	71.5	36.7	21.1
2000	70.3	35.2	13.5	70.9	35.5	17.0	71.5	35.7	20.6
2100	70.2	33.4	12.8	70.8	33.7	16.2	71.3	34.0	19.5
2200	70.1	31.8	12.2	70.6	32.1	15.4	71.3	32.4	18.6
2300	70.0	30.4	11.7	70.5	30.7	14.7	71.0	30.9	17.8
2400	69.9	29.1	11.2	70.4	29.3	14.1	70.9	29.6	17.0
2500	69.8	27.9	10.7	70.3	28.1	13.5	70.8	28.3	16.3

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	14 Diopters			16 Diopters			18 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1200	73.9	61.6	41.3	74.5	62.1	47.6	75.1	62.6	54.0
1250	73.8	59.0	39.6	74.3	59.5	45.6	74.9	59.9	51.7
1300	73.6	56.6	37.9	74.2	57.0	43.8	74.7	57.5	49.6
1350	73.4	54.4	36.5	74.0	54.8	42.0	74.5	55.2	47.6
1400	73.3	52.3	35.1	73.8	52.7	40.4	74.4	53.1	45.8
1450	73.1	50.4	33.8	73.7	50.8	39.0	74.2	51.2	44.1
1500	73.0	48.7	32.7	73.6	49.0	37.6	74.0	49.4	42.6
1550	72.9	47.0	31.5	73.4	47.4	36.3	73.9	47.7	41.1
1600	72.8	45.5	30.5	73.3	45.8	35.1	73.8	46.1	39.8
1650	72.6	44.0	29.6	73.2	44.3	34.0	73.6	44.6	38.5
1700	72.5	42.7	28.6	73.0	43.0	33.0	73.5	43.2	37.3
1750	72.4	41.4	27.8	72.9	41.7	32.0	73.4	41.9	36.2
1800	72.3	40.2	27.0	72.8	40.5	31.0	73.3	40.7	35.1
1850	72.2	39.0	26.2	72.7	39.3	30.2	73.2	40.0	34.1
1900	72.1	38.0	25.5	72.6	38.2	29.3	73.1	38.5	33.2
1950	72.0	37.0	24.8	72.5	37.2	28.5	73.0	37.4	32.3
2000	72.0	36.0	24.1	72.4	36.2	27.8	72.9	36.4	31.4
2100	71.8	34.2	22.9	72.3	34.4	26.4	72.7	34.6	30.0
2200	71.6	32.6	21.9	72.1	32.8	25.1	72.5	33.0	28.4
2300	71.5	31.1	20.9	71.9	31.3	24.0	72.3	31.5	27.1
2400	71.4	29.7	20.0	71.8	29.9	22.9	72.2	30.1	26.0
2500	71.2	28.5	19.1	71.7	28.7	22.0	72.1	28.8	24.9

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	20 Diopters			22 Diopters			24 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
1200	75.6	63.0	60.4	76.1	63.4	66.9	76.6	63.9	73.5
1250	75.4	60.3	57.7	75.9	60.7	64.0	76.4	61.1	70.3
1300	75.2	57.9	55.4	75.7	58.2	61.4	76.2	58.6	67.4
1350	75.0	55.6	53.3	75.5	55.9	59.0	76.0	56.3	64.7
1400	74.9	53.5	51.2	75.3	53.8	56.7	75.8	54.1	62.3
1450	74.7	51.5	49.4	75.2	51.8	54.7	75.6	52.1	60.0
1500	74.5	49.7	47.6	75.0	50.0	52.7	75.4	50.3	57.8
1550	74.4	48.0	46.0	74.8	48.3	50.9	75.3	48.6	55.8
1600	74.2	46.4	44.5	74.7	46.7	49.2	75.1	46.9	54.0
1650	74.1	44.9	43.0	74.5	45.2	47.6	75.0	45.4	52.3
1700	74.0	43.5	41.7	74.4	43.8	46.1	74.8	44.0	50.6
1750	73.8	42.2	40.4	74.3	42.4	44.7	74.7	42.7	49.1
1800	73.7	41.0	39.3	74.1	41.2	43.4	74.5	41.4	47.6
1850	73.6	39.8	38.1	74.0	40.0	42.2	74.4	40.2	46.2
1900	73.5	38.7	37.1	73.9	38.9	41.0	74.3	39.1	45.0
1950	73.4	37.6	36.1	73.8	37.8	39.9	74.2	38.0	43.8
2000	73.3	36.6	35.1	73.7	36.8	38.8	74.0	37.0	42.6
2100	73.1	34.8	33.4	73.5	35.0	36.9	73.8	35.2	40.4
2200	72.9	33.1	31.8	73.3	33.3	35.1	73.6	33.5	38.5
2300	72.7	31.6	30.3	73.1	31.8	33.5	73.5	31.9	36.7
2400	72.6	30.2	29.0	72.9	30.4	32.0	73.3	30.5	35.1
2500	72.4	29.0	27.8	72.8	29.1	30.8	73.1	29.2	33.6

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	10 Diopters			12 Diopters			14 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
800	74.2	92.8	44.5	75.1	93.9	54.0	75.9	94.9	63.7
820	74.1	90.4	43.3	75.0	91.4	52.6	75.8	92.4	62.0
840	74.0	88.1	42.2	74.9	89.1	51.3	75.6	90.0	60.4
860	73.9	86.0	41.2	74.8	86.9	50.0	75.5	87.8	58.9
880	73.8	83.9	40.2	74.6	84.8	48.8	75.4	85.7	57.5
900	73.7	81.9	39.3	74.5	82.8	47.6	75.3	83.6	56.1
920	73.6	80.0	38.4	74.4	80.9	46.5	75.2	81.7	54.8
940	73.5	78.2	37.3	74.3	79.1	45.5	75.1	79.8	53.6
960	73.4	76.5	36.7	74.2	77.3	44.5	75.0	78.1	52.4
980	73.4	74.9	35.9	74.1	75.6	43.5	74.9	76.4	51.3
1000	73.3	73.3	35.1	74.0	74.0	42.6	74.8	74.8	50.2
1020	73.2	71.8	34.4	74.0	72.5	41.7	74.7	73.2	49.1
1040	73.1	70.3	33.7	73.9	71.0	40.9	74.6	71.7	48.1
1060	73.0	68.9	33.0	73.8	69.6	40.0	74.5	70.3	47.2
1080	73.0	67.6	32.4	73.7	68.3	39.3	74.4	68.9	46.2
1100	72.9	66.3	31.8	73.6	67.0	38.5	74.3	67.6	45.3
1120	72.8	65.0	31.2	73.6	65.7	37.8	74.2	66.3	44.5
1140	72.8	63.8	30.6	73.5	64.5	37.1	74.2	65.0	43.6
1160	72.7	62.7	30.0	73.4	63.3	36.4	74.1	63.9	42.9
1180	72.6	61.6	29.5	73.4	62.2	35.8	74.0	62.7	42.1
1200	72.6	60.5	28.9	73.3	61.1	35.1	73.9	61.6	41.3

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	16 Diopters			18 Diopters			20 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
800	76.6	95.8	73.5	77.3	96.6	83.4	78.0	97.5	93.4
820	76.5	93.3	71.5	77.2	94.1	81.2	77.8	94.9	91.0
840	76.4	90.9	69.7	77.0	91.7	79.1	77.7	92.5	87.4
860	76.2	88.6	68.0	76.9	89.4	77.1	77.5	90.2	86.4
880	76.1	86.5	66.3	76.8	87.2	75.2	77.4	87.9	84.3
900	76.0	84.4	64.7	76.6	85.1	73.5	77.2	85.8	82.3
920	75.9	82.5	63.2	76.5	83.2	71.7	77.1	83.8	80.4
940	75.8	80.6	61.8	76.4	81.3	70.1	77.0	81.9	78.5
960	75.6	78.8	60.4	76.3	79.4	68.5	76.9	80.1	76.7
980	75.5	77.1	59.1	76.2	77.7	67.0	76.8	78.3	75.1
1000	75.4	75.4	57.8	76.0	76.0	65.6	76.6	76.6	73.5
1020	75.3	73.8	56.6	75.9	74.4	64.2	76.5	75.0	71.9
1040	75.2	72.3	55.4	75.8	72.9	62.9	76.4	73.5	70.4
1060	75.0	70.8	54.3	75.7	71.4	61.6	76.3	72.0	69.0
1080	75.0	69.5	53.3	75.6	70.0	60.4	76.2	70.6	67.6
1100	75.0	68.1	52.2	75.5	68.7	59.2	76.1	69.2	66.3
1120	74.9	66.8	51.3	75.4	67.4	58.1	76.0	67.9	65.1
1140	74.8	65.6	50.3	75.4	66.1	57.0	75.9	66.6	63.8
1160	74.7	64.4	49.4	75.3	64.9	56.0	75.8	65.4	62.6
1180	74.6	63.2	48.5	75.2	63.7	55.0	75.7	64.2	61.5
1200	74.5	62.1	47.6	75.1	62.6	54.0	75.6	63.0	60.4

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	22 Diopters			24 Diopters			26 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
800	78.6	98.2	104	79.2	99.0	114	79.8	99.7	124
820	78.4	95.7	101	79.0	96.4	111	79.6	97.1	121
840	78.3	93.2	98.3	78.9	93.9	108	79.4	94.6	118
860	78.1	90.8	95.8	78.7	91.5	105	79.2	92.2	115
880	78.0	88.6	93.4	78.6	89.3	103	79.1	89.9	112
900	77.8	86.5	91.2	78.4	87.1	100	78.9	87.7	109
920	77.7	84.5	89.0	78.3	85.1	97.7	78.8	85.6	107
940	77.6	82.5	87.0	78.1	83.1	95.6	78.6	83.7	104
960	77.4	80.7	85.1	78.0	81.2	93.4	78.5	81.8	102
980	77.3	78.9	83.2	77.9	79.4	91.4	78.4	80.0	99.6
1000	77.2	77.2	81.4	77.7	77.7	89.4	78.2	78.2	97.5
1020	77.1	75.6	79.7	77.6	76.1	87.5	78.1	76.6	95.4
1040	77.0	74.0	78.0	77.5	74.5	85.7	78.0	75.0	93.4
1060	76.8	72.5	76.4	77.4	73.0	83.9	77.9	73.4	91.5
1080	76.7	71.1	74.9	77.2	71.5	82.3	77.7	72.0	89.7
1100	76.6	69.7	73.5	77.1	70.1	80.7	77.6	70.6	88.0
1120	76.5	68.3	72.0	77.0	68.8	79.1	77.5	69.2	86.2
1140	76.4	67.0	70.7	76.9	67.5	77.6	77.4	67.9	84.6
1160	76.3	65.8	69.4	76.8	66.2	76.2	77.3	66.6	83.0
1180	76.2	64.6	68.1	76.7	65.0	74.8	77.2	65.4	81.5
1200	76.1	63.4	66.9	76.6	63.9	73.5	77.1	64.2	80.1

TABLE 7 cont'd.

BI - CONVEX									
Freq. KC	28 Diopters			30 Diopters			32 Diopters		
	ft	t	t/R	ft	t	t/R	ft	t	t/R
800	80.3	100	135	80.8	101	145	81.4	102	156
820	80.1	97.7	131	80.7	98.4	141	81.2	99.0	152
840	78.0	95.2	128	80.5	95.8	138	81.0	96.4	148
860	79.8	92.8	125	80.3	93.4	134	80.8	93.9	144
880	79.6	90.5	121	80.1	91.0	131	80.6	91.6	140
900	79.5	88.3	119	80.0	88.8	128	80.4	89.4	137
920	79.3	86.2	116	79.8	86.7	125	80.3	87.2	134
940	79.2	84.2	113	79.6	84.7	122	80.1	85.2	131
960	79.0	82.3	110	79.5	82.8	119	80.0	83.3	128
980	78.9	80.5	108	79.3	81.0	116	79.8	81.4	125
1000	78.7	78.7	106	79.2	79.2	114	79.7	79.7	122
1020	78.6	77.0	103	79.1	77.5	111	79.5	78.0	120
1040	78.5	75.4	101	78.9	75.9	109	79.4	76.3	117
1060	78.3	73.9	99.2	78.8	74.3	107	79.2	74.8	115
1080	78.2	72.4	97.2	78.7	72.8	105	79.1	73.2	112
1100	78.1	71.0	95.3	78.6	71.4	103	79.0	71.8	110
1120	78.0	69.6	93.4	78.4	70.0	101	78.9	70.4	108
1140	77.9	68.3	91.7	78.3	68.7	98.8	78.7	69.1	106
1160	77.8	67.0	90.0	78.2	67.4	96.9	78.6	67.8	104
1180	77.6	65.8	88.3	78.1	66.2	95.1	78.5	66.5	102
1200	77.5	64.6	86.7	78.0	65.0	93.4	78.4	65.3	100

APPENDIX C

CURVATURE CONVERSION TABLE

The spherical laps commonly employed in the lens-making industry are specified in terms of "diopters". This is the term normally used to specify the power of a lens -- the diopter number being the reciprocal of the focal length in meters. In this case, it is assumed that the lens is of plano-convex configuration and made of glass with an index of refraction of 1.530. Thus, the radius of curvature in millimeters will be 530 divided by the diopter number. In table 8, the diopter numbers of the laps commonly used in the industry, with the corresponding radius of curvature and the reciprocal of the radius in meters are listed.

TABLE 8

UNITS OF CURVATURE

CONVERSION TABLE

Diopters	Reciprocal Meters	Radius of Curvature in millimeters	Radius of Curvature in inches
.125	0.24	4240.00	166.93
.250	0.47	2120.00	83.46
.375	0.71	1413.33	55.64
.500	0.94	1060.00	41.73
.625	1.18	848.00	33.38
.750	1.42	706.67	27.82
.875	1.65	605.71	23.86
1.000	1.89	530.00	20.87
1.125	2.12	471.11	18.55
1.250	2.36	424.00	16.69
1.375	2.59	385.45	15.18
1.500	2.83	353.33	13.91
1.625	3.07	326.15	12.84
1.750	3.30	302.86	11.92
1.875	3.54	282.67	11.13
2.000	3.77	265.00	10.433
2.125	4.01	249.41	9.82
2.250	4.25	235.56	9.27
2.375	4.48	223.16	8.79
2.500	4.72	212.00	8.35
2.625	4.95	201.90	7.95
2.750	5.19	192.73	7.59
2.875	5.42	184.35	7.26
3.000	5.66	176.67	6.96
3.125	5.90	169.60	6.68
3.250	6.13	163.08	6.42
3.375	6.37	157.04	6.18
3.500	6.60	151.43	5.96
3.625	6.84	146.21	5.76
3.750	7.08	141.33	5.56
3.875	7.31	136.77	5.38
4.000	7.55	132.50	5.22
4.125	7.78	128.48	5.06
4.250	8.02	124.71	4.91
4.375	8.25	121.14	4.77
4.500	8.49	117.78	4.64
4.625	8.73	114.59	4.51
4.750	8.96	111.58	4.39
4.875	9.20	108.72	4.28
5.000	9.43	106.00	4.17
5.125	9.67	103.41	4.07
5.250	9.91	100.95	3.97
5.375	10.14	98.60	3.88
5.500	10.38	96.36	3.79

TABLE 8 cont'd.

UNITS OF CURVATURE

CONVERSION TABLE

Diopters	Reciprocal Meters	Radius of Curvature in millimeters	Radius of Curvature in inches
5.625	10.61	94.22	3.71
5.750	10.85	92.17	3.63
5.875	11.08	90.21	3.55
6.000	11.32	88.33	3.48
6.125	11.56	86.53	3.41
6.250	11.79	84.80	3.34
6.375	12.03	83.14	3.27
6.500	12.26	81.54	3.21
6.625	12.50	80.00	3.15
6.750	12.73	78.52	3.09
6.875	12.97	77.09	3.04
7.000	13.20	75.71	2.98
7.125	13.44	74.39	2.93
7.250	13.68	73.10	2.88
7.375	12.93	71.86	2.83
7.500	14.15	70.67	2.78
7.625	14.38	69.51	2.74
7.750	14.62	68.39	2.69
7.875	14.85	67.30	2.65
8.00	15.09	66.25	2.60
8.250	15.57	64.24	2.53
8.500	16.04	62.35	2.45
8.750	16.51	60.57	2.38
9.000	17.00	58.89	2.32
9.250	17.45	57.30	2.26
9.500	17.92	55.79	2.20
9.570	18.40	54.36	2.18
10.	18.87	53.00	2.09
10.5	19.81	50.48	1.99
11	20.75	48.18	1.90
11.5	21.70	46.09	1.81
12	22.64	44.17	1.74
12.5	23.58	42.40	1.67
13	24.52	40.77	1.60
13.5	25.47	39.26	1.54
14	26.42	37.86	1.49
14.5	27.36	36.55	1.44
15	28.30	35.33	1.39
16	30.19	33.12	1.30
17	32.08	31.18	1.23
18	33.96	29.44	1.16
19	35.85	27.89	1.10
20	37.73	26.50	1.04

TABLE 8 cont'd.

UNITS OF CURVATURE

CONVERSION TABLE

Diopters	Reciprocal Meters	Radius of Curvature in millimeters	Radius of Curvature in inches
21	39.62	25.24	0.994
22	41.51	24.09	0.948
23	43.40	23.04	0.907
24	45.28	22.08	0.869
25	47.17	21.20	0.835
26	49.06	20.38	0.802
27	50.94	19.63	0.773
28	52.83	18.93	0.745
29	54.72	18.28	0.720
30	56.00	17.67	0.695
31	58.49	17.10	0.673
32	60.38	16.56	0.652
33	62.26	16.06	0.632
34	64.15	15.59	0.614
35	66.04	15.14	0.596
36	67.92	14.72	0.580
37	69.81	14.32	0.564
38	71.70	13.95	0.549
39	73.58	13.59	0.535
40	75.47	13.25	0.522